Lecture 24

Markets, Mechanisms and Machines

David Evans and Denis Nekipelov

Privacy and incentives

- In economics literature privacy is studied from incentive perspective
  - How does a game change if one player has more information than the other?
- “Privacy regime” corresponds to how much information is shared between the players
- Typically this is context-specific
- Only information that is directly relevant for transaction is considered
- Value of privacy can be measured by changes social welfare induced by behavior in “private” vs “non-private” settings

Privacy and incentives

- Private information corresponds to “types” or valuations in auctions
- Usually consider asymmetric settings: one player (principal) who wants to exploit private information, another player (agent) who has private information but does not want it to be exploited
- Principal and agents move sequentially: principal gives a “contract” to agent, agent chooses action based on “terms”
- Privacy measured by amount of information that leaks from principal to agent
- Natural way of summarizing information transfer: probability distribution (beliefs)

Dynamic Bayesian Games

Many practical environments are dynamic

- Nature assigns types, players act (sequentially)

Need extension of Nash and subgame Nash equilibria to Bayesian settings

Perfect Bayesian Equilibrium (PBE)

- Two kinds of players:informed and uninformed (firms and workers, entrants and incumbents).
- Uninformed player tries to infer informed player's private information from her behavior to choose action
- Informed player takes inference into account when choosing her action

Dynamic Bayesian Games

Uninformed players realize that informed ones change their action knowing that it can be used "against" them

Need to take that into account to infer properly

Impossible to derive best responses without knowing inference rule

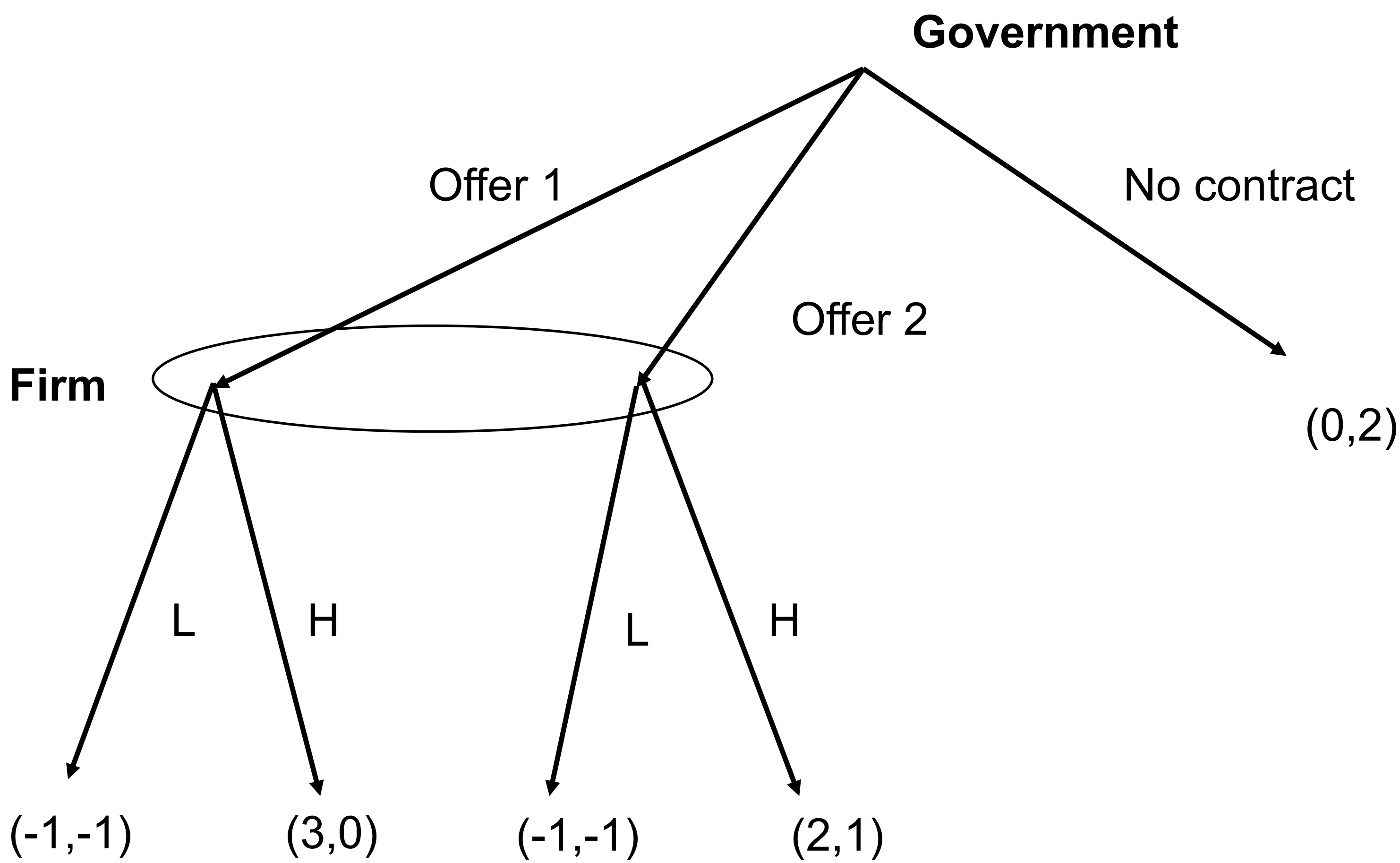
PBE has a built-in concept of inference called beliefs (based on Bayesian updating)

Best responses are evaluated using that concept

Example: procurement contract

- Government can offer (or not) a construction firm a contract
- Goal of the government: guarantee on time project completion
  - E.g infrastructure project
- Government can use two different on-time completion incentive payouts that are only revealed after job is complete
- Firm can exert (high or low) effort and complete project on time (or not)
- Goal of the firm: maximize expected utility

Example: procurement contract



Example: procurement contract

- The game has only one subgame
- SPNE are NE of the bi-matrix game:

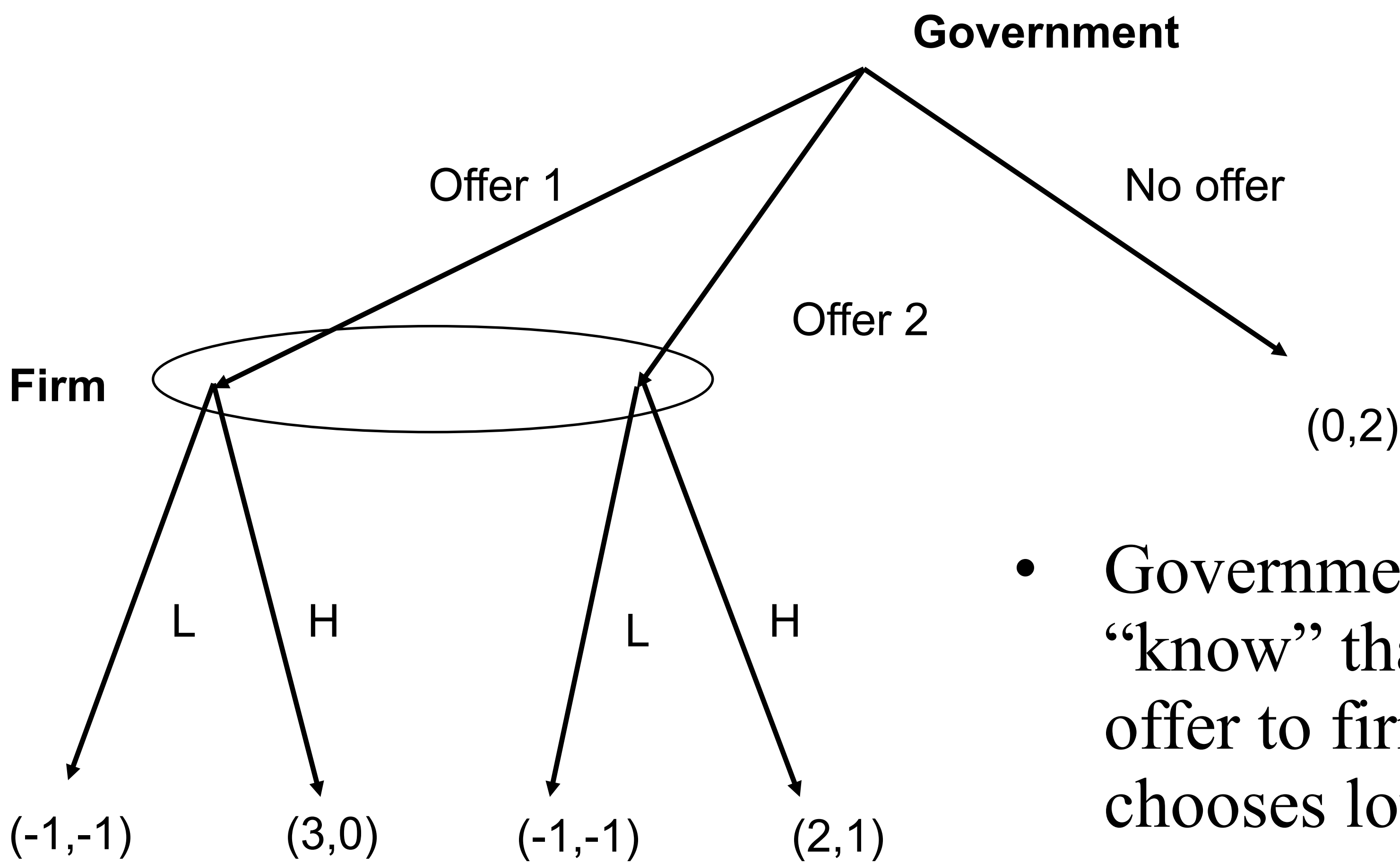
	L	H
Offer 1	(-1,-1)	(3,0)
Offer 2	(-1,-1)	(2,1)
No offer	(0,2)	(0,2)

Example: procurement contract

- Two pure-strategy NE
  - (Offer 1, H)
  - (No offer, L)

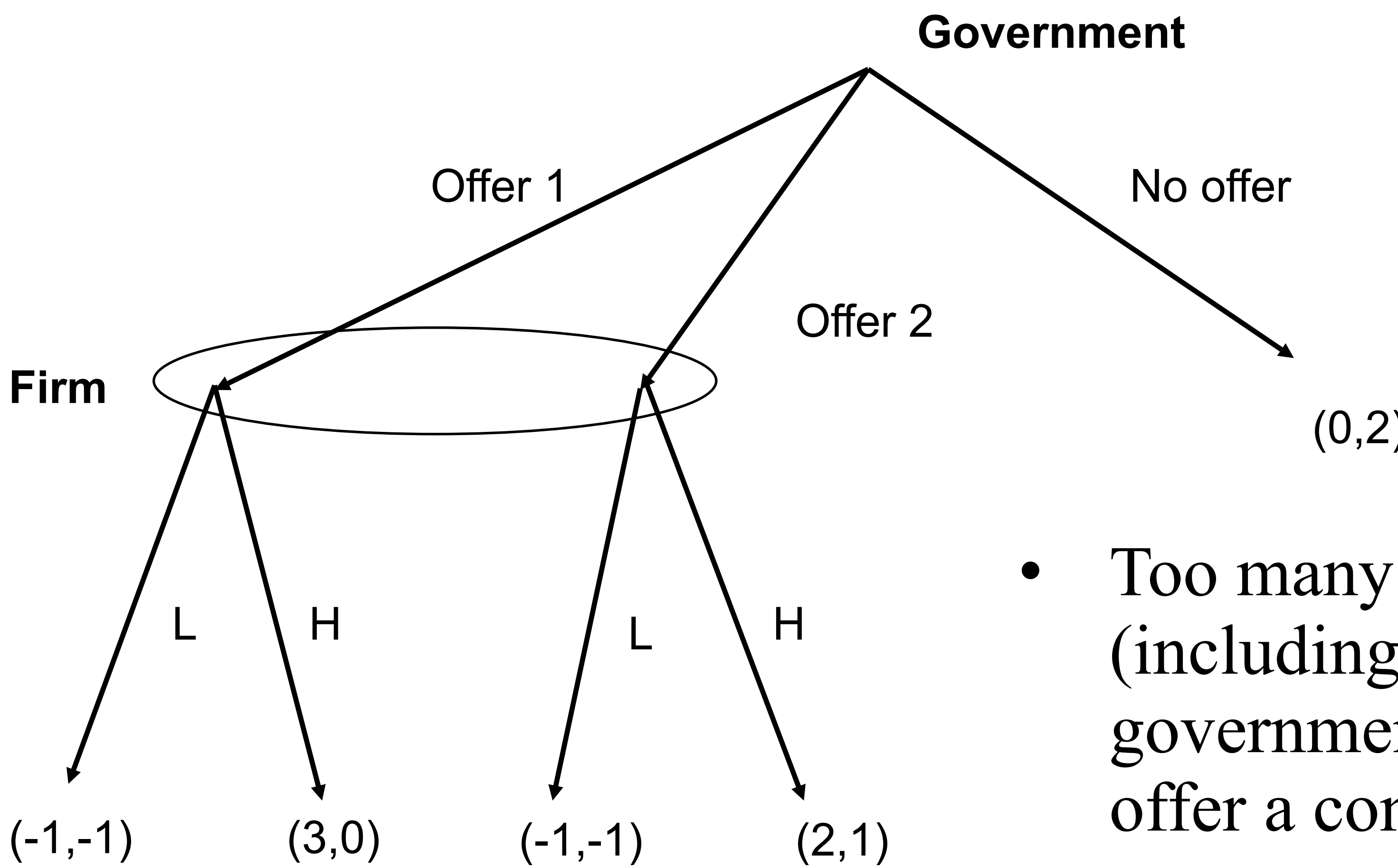
	L	**H**
Offer 1	(-1,-1)	**(3,0)**
Offer 2	(-1,-1)	(2,1)
No offer	**(0,2)**	(0,2)

Example: procurement contract



- Government should "know" that if it gives offer to firm, firm never chooses low effort!

Example: procurement contract



- Too many SPNE (including the one where government does not offer a contract)!

Example: procurement contract

- Solution: introduce beliefs as part of solution concept
- Beliefs correspond to “educated guess” (in the form of probability distribution) of one player regarding how other player acts
- In our example, firm should never exert low effort regardless of what it beliefs the government has chosen for the incentive contract

Perfect Bayesian Equilibrium

Beliefs of player i are a conditional distribution over the elements of the information set i is in, given player i is in that information set: $P_i(v \mid S)$ for v in S

Belief in static Bayesian games (e.g. auctions) is $P_i(v_{-i} \mid v_i)$ (distribution of profile of types of opponent given own type)

Note: information set in static Bayesian game has only one element v_i

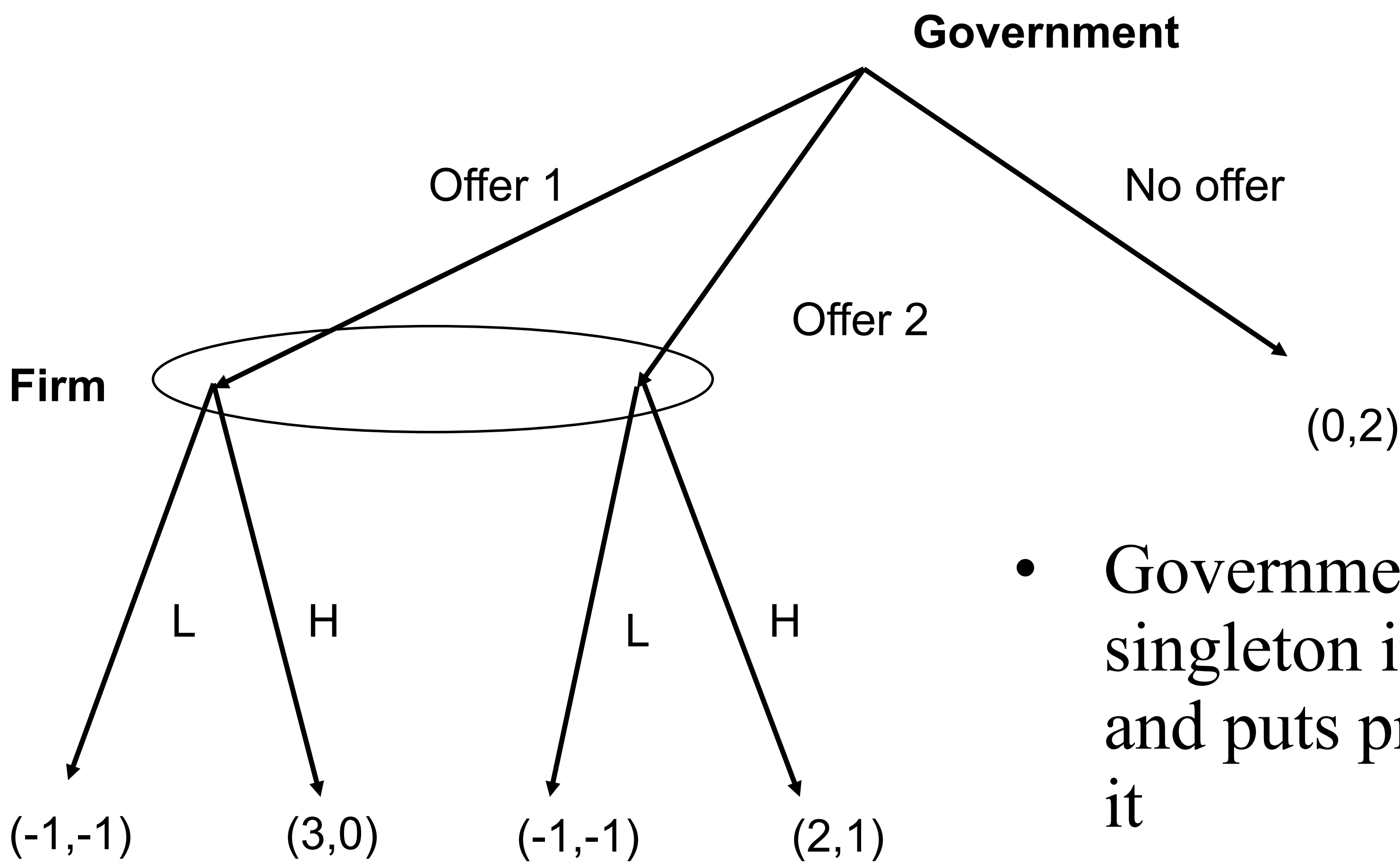
Perfect Bayesian Equilibrium

Perfect Bayesian equilibrium (PBE) strengthens subgame perfection by requiring two elements:

a complete strategy for each player *i* (mapping from info. sets to mixed actions)

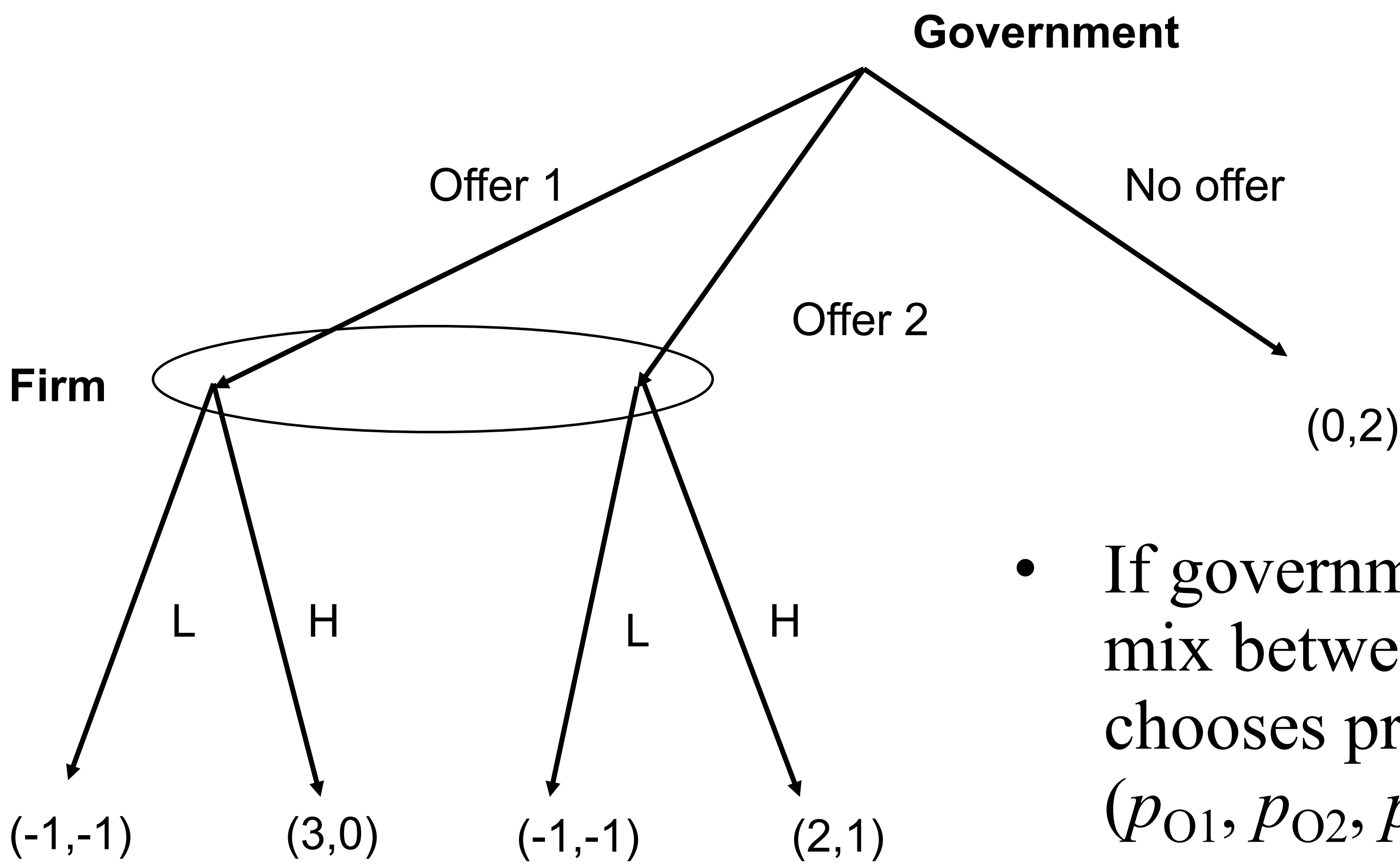
beliefs for each player i: $P_i(v \mid S)$ for all information sets S which player *i* can reach

Example: procurement contract



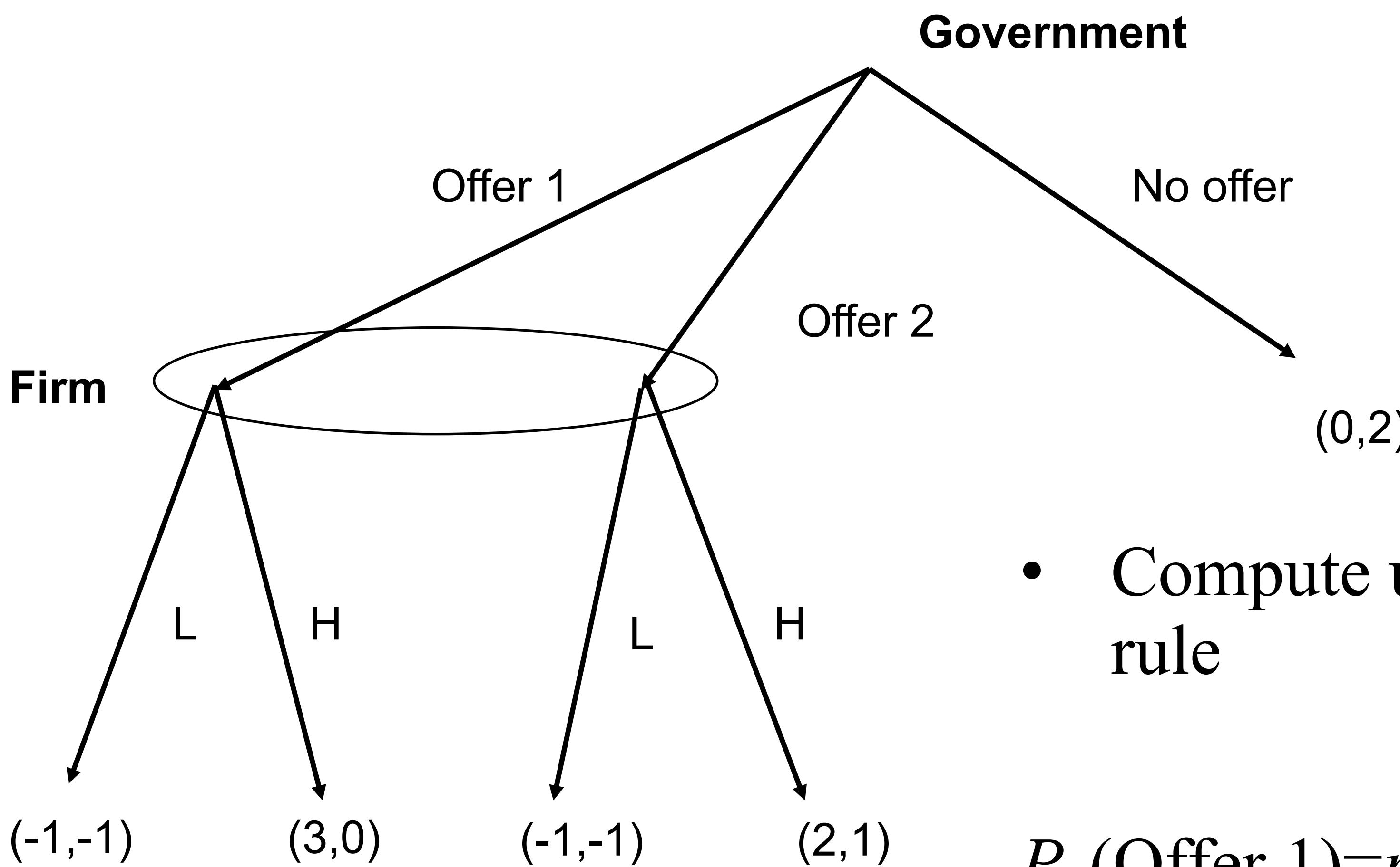
- Government has a singleton information set and puts probability 1 on it

Example: procurement contract



- If government decides to mix between options and chooses probabilities (p_{O1}, p_{O2}, p_{NO})
- Firm has to guess which node it is in

Example: procurement contract



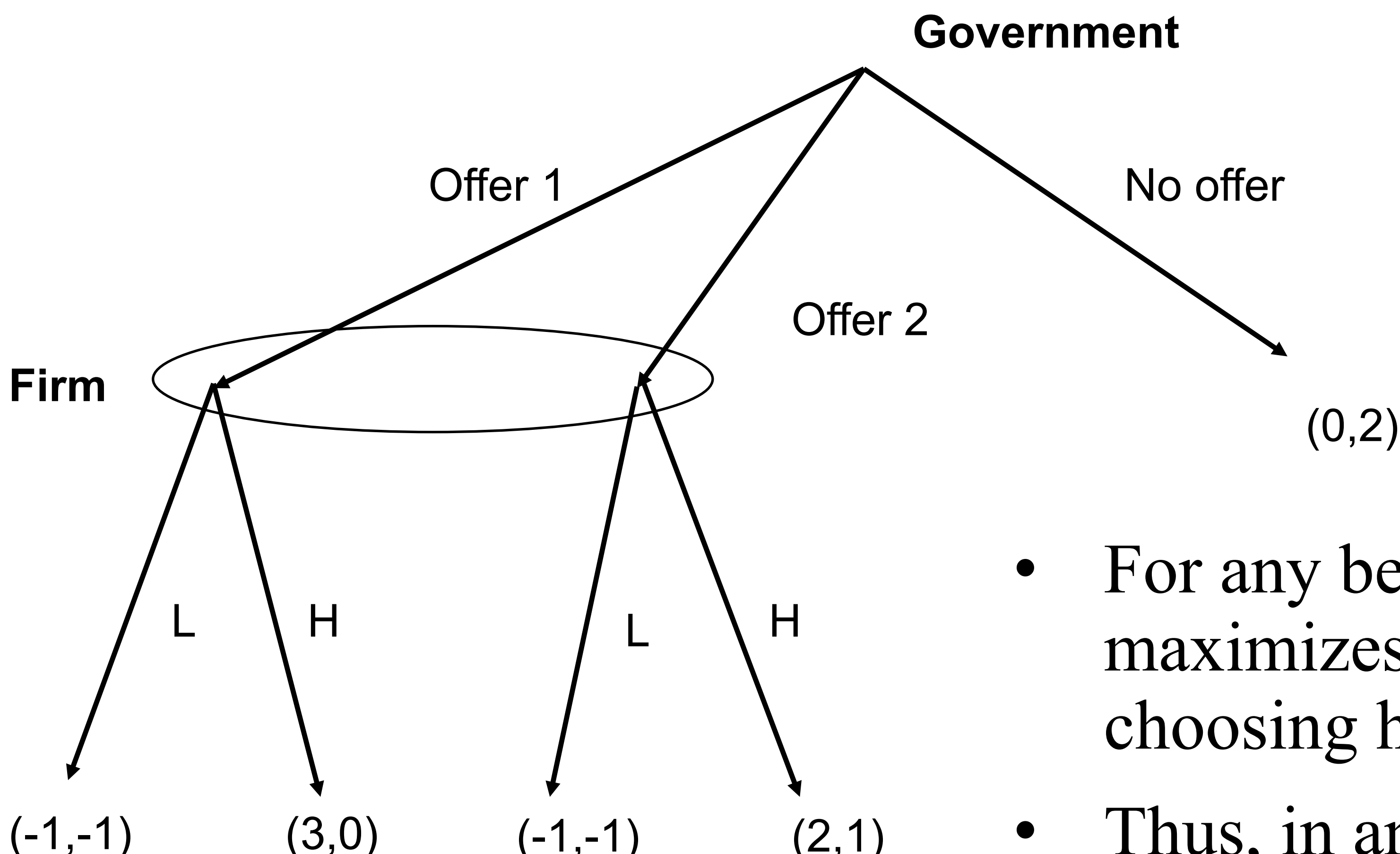
- Compute using Bayes' rule

$P_F(\text{Offer 1}) = p_{O1} / (p_{O1} + p_{O2})$

Perfect Bayesian Equilibrium

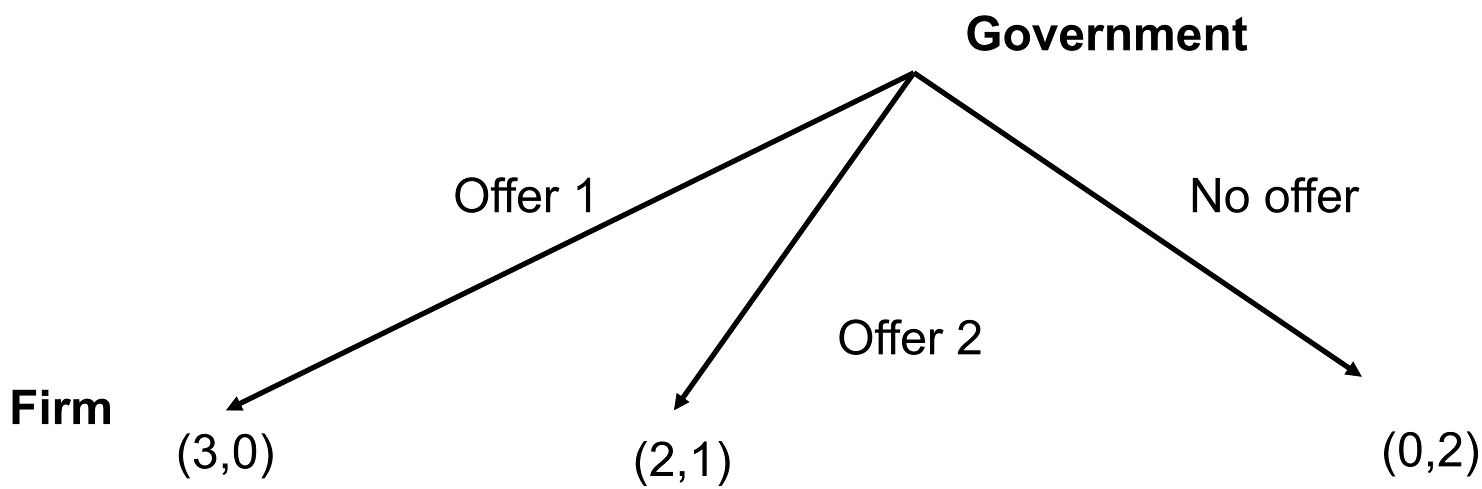
- PBE formalizes the concept of "guessing" using available information
- We mandate the "guessing" to occur based on the Bayes rule
- In equilibrium beliefs should be correct: Bayes rule-derived probabilities are equal to actual probabilities of occurrence of information sets
- Players act to maximize payoffs
    - Player i's strategy $s_i(\cdot)$ is such that in any information set h of player i,
    - $s_i(h)$ maximizes i's expected payoff, given her beliefs and others' strategies

Example: procurement contract



- For any beliefs, firm maximizes it's payoff by choosing high effort
- Thus, in any PBE firm should play H

Example: procurement contract



- In PBE government chooses Offer 1 in its information set
- PBE is unique: SPNE (No offer, L) was eliminated

Perfect Bayesian Equilibrium

- In PBE with private types of players principal is interested in “revealing” those types
- PBE where agents with different types act differently is called separating PBE
- PBE where groups of agents with different types choose the same action is called pooling PBE
- “Privacy regime” is determined by whether PBE separates types

Control of information

- Spence (1973): “The lemon market”
  - To avoid market collapse individuals may engage in costly “signaling” to reveal their type
- In separating equilibrium with signaling, individuals with different types choose different signals
- Principal can treat different types differently
- Privacy allows individuals not to waste effort on signaling
- Gottlieb and Smetters (2011): 9 out of 15 top MBA programs in the US do not disclose student grades to employers

Control of information

- Simple model: Ability of MBA student $\theta \in [0,1]$ produces grade g with effort cost g/θ
- When student graduates θ is her productivity at work
- With public grades, offered wage will depend on g
- When grades are not public, employers have to pay the same wage to all MBA graduates

Control of information

- Utility of graduate

  $U(w, g, \theta) = w - g/\theta$
- Profit of the firm is $\theta - w$
- Assume that θ takes values on [0,1]
- This principal-agent setting of a sequential game:
  1. MBA graduate makes decision to exert effort by choosing g
  2. Firms make competitive offers w
  3. MBA graduate accepts or rejects it

Control of information

- Find grade-dependent wage $w(g)$
  - MBA student chooses effort (expressed in grade g) to maximize utility $U(w(g), g, \theta)$ with respect to
    - FOC: $w'(g)=1/\theta$, which implicitly defines $g(\theta)$
  - Firms make competitive offers $w(g)$
    - Since firms know $w(g)$, they know mapping $g(\theta)$
    - Thus firm can infer θ from observing grade g
  - Competitive offer is then $w=\theta$
  - This means that $w(g)= \theta = 1/w'(g)$
  - Solve differential equation to get

$$w(g) =(2g)^{1/2} \text{ (calibration } w(0) =0)$$

Control of information

In separating equilibria

- Students with different abilities choose different effort
- In this equilibrium $g^*(\theta)=\theta^2/2$ (students with higher ability earn higher grades)
- Equilibrium payment $w^*(g)=\theta$ and utility $U^*(\theta)=\theta/2$

- In "grade privacy" regime firms offer uniform wage w_U =E[θ]
  - None of the students exert effort
  - This is a "pooling" equilibrium
- Grade privacy is optimal if $U^*(\theta)<$ E[θ], i.e. E[θ]>1/2
  - MBA students have to be "selectively smart"

Markets for information

- Varian (1997): consumers may suffer privacy costs when "principal" knows too little information about them
  - It limits the ability of principal to customize the product
- At the same time, consumers have opposite incentive to not share too much information
  - Consumers want to limit price discrimination
- Consumer may rationally decide to share personal information with principal
  - However, she does not control information after it is communicated to principal
  - Principal may sell consumer's data to third parties that would use it for their purposes
  - Third parties create externality for information sharing

Markets for information

- Consider market for particular product
- Consumers are "infinitesimal" (cannot influence market price individually) with valuations uniformly distributed on [0,1]
- Market served by monopolist with zero production cost
  - This is a normalization
- Without market for information
  - Firm's profit from offering price p: $p(1-p)$ (fraction of consumers with values below the price is $1-p$)
  - Monopolist sets the price $p_M=1/2$
  - Firm's optimal profit is ¼
  - Aggregate consumer surplus is $(p-p^2/2)|^1_{0.5}=1/8$

Markets for information

- Suppose that each consumer has verifiable information (e.g., place of residence or employment) perfectly correlated with her valuation for product.
- Firm first makes offer to pay $r \geq 0$ to consumer revealing her information
- And uses information to make personalized price offers $p^*(v)$ to consumers who sold their information and common price p to everyone else
- Equilibrium concept: perfect Bayesian equilibrium (PBE)
  - Bayes-Nash equilibrium concept applied to sequential settings
  - Requires players to form beliefs regarding opponent types

Markets for information

- In PBE
  - Firm offers $r = 0$ for information.
  - All consumers reveal their valuations,
  - Firm sets $p^*(v)=v$ and $p = 1$.
- Note that high-value consumers will be served in any case
- Low value consumers will be served if they revealed their values
- Marginal anonymous consumer makes no surplus and reveals her valuation for an arbitrarily small payment
- This means that there are now marginal consumers in equilibrium: everyone reveals their values

Markets for information

- All consumers are served in equilibrium
- Social welfare generated is $(p-p^2/2)|^1_0=1/2$
- This equilibrium is efficient
- However, consumer surplus is now 0
- Even though consumers had ownership of their information, unregulated market for information transferred all their surplus to the monopolist

Information and price discrimination

- Dynamic settings can be more realistic
- Firm that sells product in many period can learn about valuations of consumers for product given that they did not purchase at a given price
- This allows firms to engage in intertemporal price discrimination
- The extent of price discrimination is further amplified when some consumers are naïve and do not anticipate that information they reveal to the firm in a given period will be used by firm for pricing in future periods

Consumer tracking

- Two-period market
- Population of n consumers with unit demand in each period
- Half of consumers have valuation 1 (high valuation) in both periods
- The other half have valuations $\lambda \in (0,1/2)$ (low valuation) in both periods.
- Each consumer's valuation is privately known
- Product is sold by a monopolist with production cost normalized to 0

Consumer tracking

- Consumers and firm are risk neutral and do not use time discounting
- Common knowledge that monopolist has tracking technology (cookies, browser fingerprints) with which it can recall whether (and at what price) consumer purchased the good in first period
- Monopolist can use this information to make personalized price offers to consumers in second period.

Consumer tracking

- PBE characterization:
- Monopolist makes first-period price offers
- $p_1 = 1$ to all consumers and second period offers $p_2 = 1$ to all consumers regardless of their purchase histories.
- low-valuation consumers never purchase the good
- High-valuation consumer purchases with probability 1 in the second period but purchases $(1\text{-}2\ \lambda)/(1\text{-}\ \lambda)<1$ in the first period
- This makes monopolist indifferent between offering $p_2 = 1$ and $p_2 = \lambda$ following the first period rejection

Consumer tracking

- If monopolist could publicly commit not to use tracking technology then price offers would be the same,

  $p_1 = p_2 = 1$
- BUT high-valuation consumers would accept with probability 1 in first period
- Rejections in this setting could never induce lower second-period prices
- Tracking technology leads to strategic first-period rejections by high-valuation consumers
- This is welfare suboptimal with a loss of surplus of $n\lambda/(1- \lambda)$