Lecture 15

Markets, Mechanisms and Machines

David Evans and Denis Nekipelov

Online advertising

- Many online platforms use economic mechanisms to determine market outcomes
  - Online advertising
  - Online search ”verticals” (travel, jobs, real estate)
  - Online retail

Features of online marketplaces

- Dynamic environments
  - Composition of the marketplace, demand and supply change
- Limited information available to market designers
  - Have to rely on inference to make allocations and predict behavior of players
- Difficult for bidders to properly form expectations about the future
  - Need to rely on adaptive learning
  - May have much more information relative to static environments

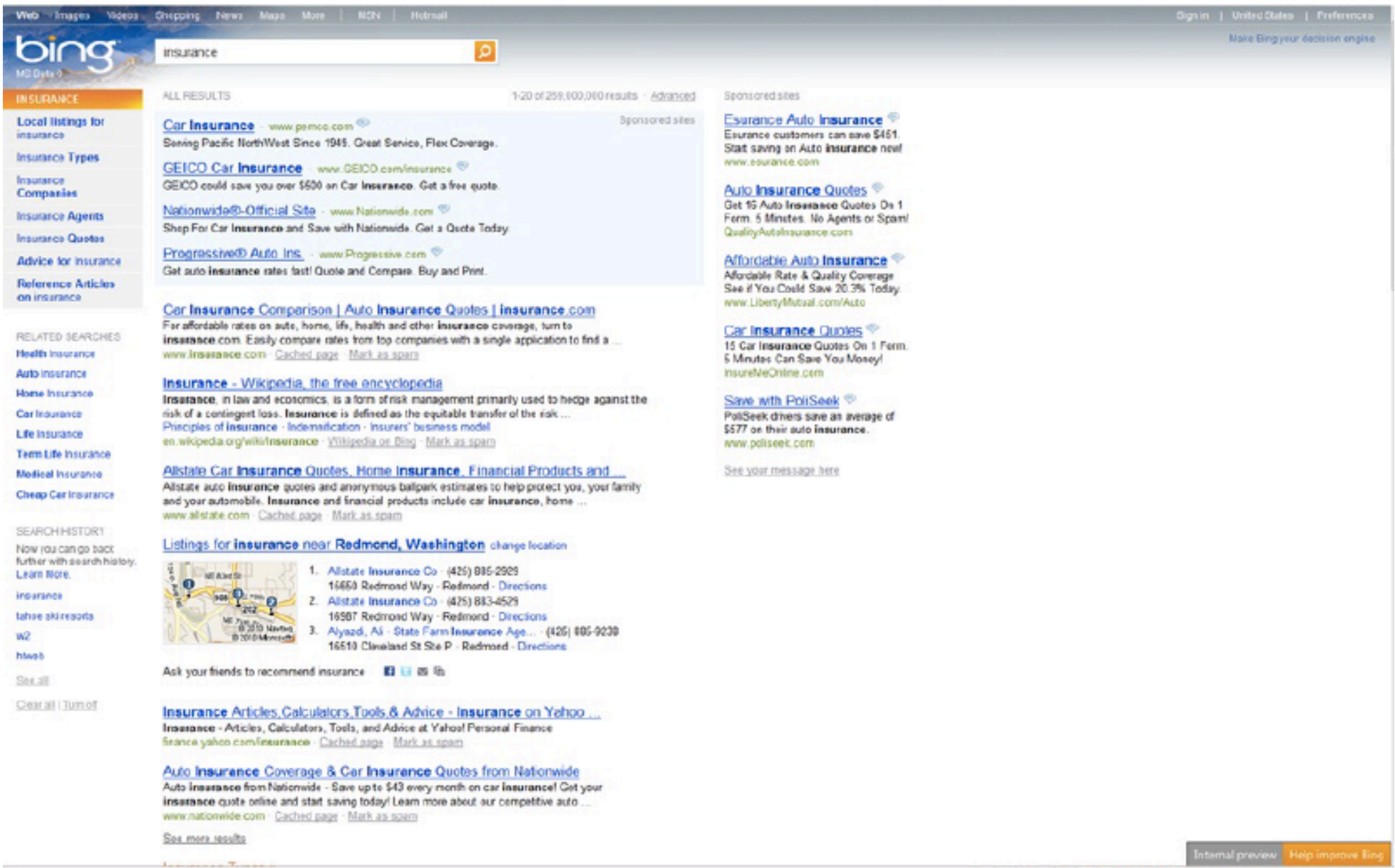
Selling advertising slots

- Traditional model: direct negotiation with advertisers
  - **Pro's :** Predictable outcomes and allocations; direct relationship between platform and advertisers
  - **Con's :** Advertiser behavior cannot respond to changing demand; hard to change prices and allocations if tastes or volumes change
- Market-based model: auction or other similar market mechanism that "automates" pricing and allocations
  - **Pro's :** Responds to changing demand through competition of advertisers; more inclusive for new or smaller advertisers
  - **Con's :** Market participants need to know how to play; harder to predict the market

Prediction of online advertising marketplaces in equilibrium

- **Assumption**: bidders know their objective functions and can optimize them.
- **Equilibrium**: bidder's bid must be best response to competing bid distribution.
- **Observation**: competing bids distribution is observed in data.
- Approach to recover primitives:
  1. given bid distribution, solve for bid strategy
  2. invert bid strategy to get bidder's value for item from bid.
- **Solution**: using values predict outcome of new mechanism

Prediction of online advertising marketplaces in equilibrium



Auction for keywords

- The ads are allocated and priced for each user query
- Pricing and allocation decisions are combined and fully automated by an "auction":
  - Real-time
  - Pay per click
  - Score-weighted
  - Generalized second price (GSP)
  - With possible reserve prices and thresholds

Allocating and pricing multiple heterogeneous objects

Need to allocate and price multiple heterogenous objects (slots) at the same time with little computation

N-pirate problem

- Need to allocate and price multiple heterogenous objects (slots) at the same time with little computation
- Imagine *N* pirates that need to split a heterogeneous treasure
- If one pirate is accused of claiming an unfair share of the treasure, he gets thrown overboard
- Easiest to imagine solution for 2 pirates and generalize
- Leads to the envy free refinement of Nash equilibrium in multi-unit auction: no bidder benefits from switching bid with any other bidder

GSP auction

- Components of the auction
  - Bids of bidders
  - Payment per click
  - Model for user clicks (multiplicative)
    - Position effect
    - Advertiser effect (score)
  - GSP payment and allocation rule:
  - bidders ranked by score-weighted bids
  - expected payment of each bidder proportional to score-weighted bid of the bidder ranked below

GSP auction

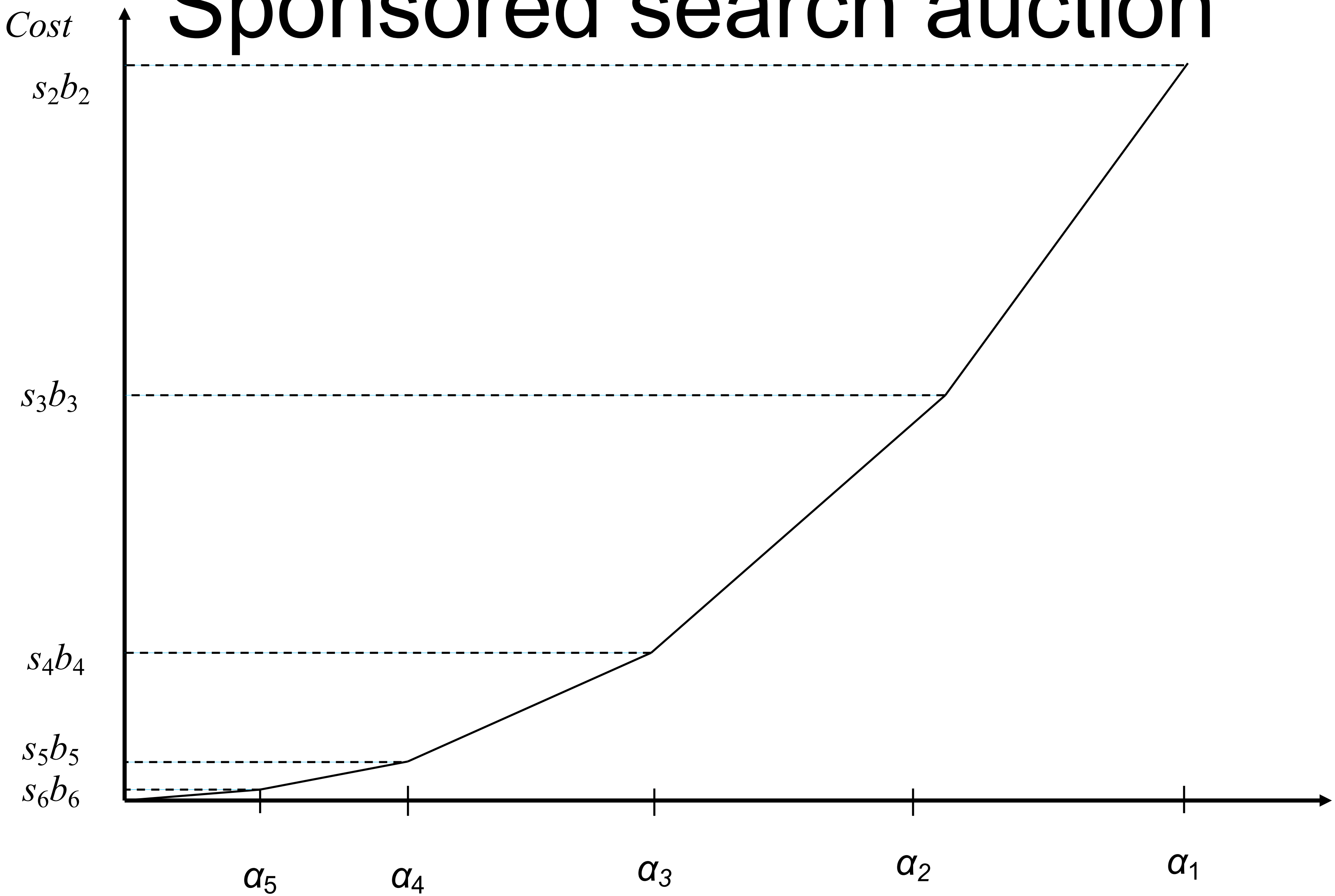
- Example: 4 bidders, 3 slots, reserve price R

Bid	*Score*	*Slot clickthrough rate*	*Score-weighted bid*	*Price*	*Expected payment per search*
b_1	s_1	α_1	$s_1 b_1$	$s_2 b_2 / s_1$	$\alpha_1 s_2 b_2$
b_2	s_2	α_2	$s_2 b_2$	$s_3 b_3 / s_3$	$\alpha_2 s_3 b_3$
b_3	s_3	α_3	$s_3 b_3$	$\min\{s_4 b_4 / s_3, R\}$	$\alpha_3 \min\{s_4 b_4, s_3 R\}$
b_4	s_4	0	$s_4 b_4$	0	0

Sponsored search auction

- Assume that bidders can interact with high frequency: by changing bids sufficiently can learn own and opponent scores as well as bids
- This game has complete information
- Moreover, with high frequency assumption can focus on the *ex-post refinement*: bidders are happy with how they bid after they learned what their opponents bids
- Best response constructed by considering incremental cost per click: how much more bidder *i* needs to pay to get an extra click?

Sponsored search auction



- Cost of bidder *i* as a function of her score-weighted bid
- It is a convex function: look at the marginal cost

Sponsored search auction

- In a Nash equilibrium with ex-post refinement

$$\begin{cases} \alpha_i\left(v_i - \frac{s_k b_k}{s_i}\right) \geq \alpha_l\left(v_i - \frac{s_m b_m}{s_i}\right), & l = m-1 \geq i = k-1 \\ \alpha_i\left(v_i - \frac{s_k b_k}{s_i}\right) \geq \alpha_l\left(v_i - \frac{s_m b_m}{s_i}\right), & i+1 = k \geq m = l+1. \end{cases}$$

or

$$\min_{l<i} \frac{s_l b_l \alpha_i - s_{i+1} b_{i+1} \alpha_l}{\alpha_l - \alpha_i} \geq s_i v_i \geq \max_{l>j} \frac{s_{i+1} b_{i+1} \alpha_j - s_{l+1} b_{l+1} \alpha_l}{\alpha_i - \alpha_l}.$$

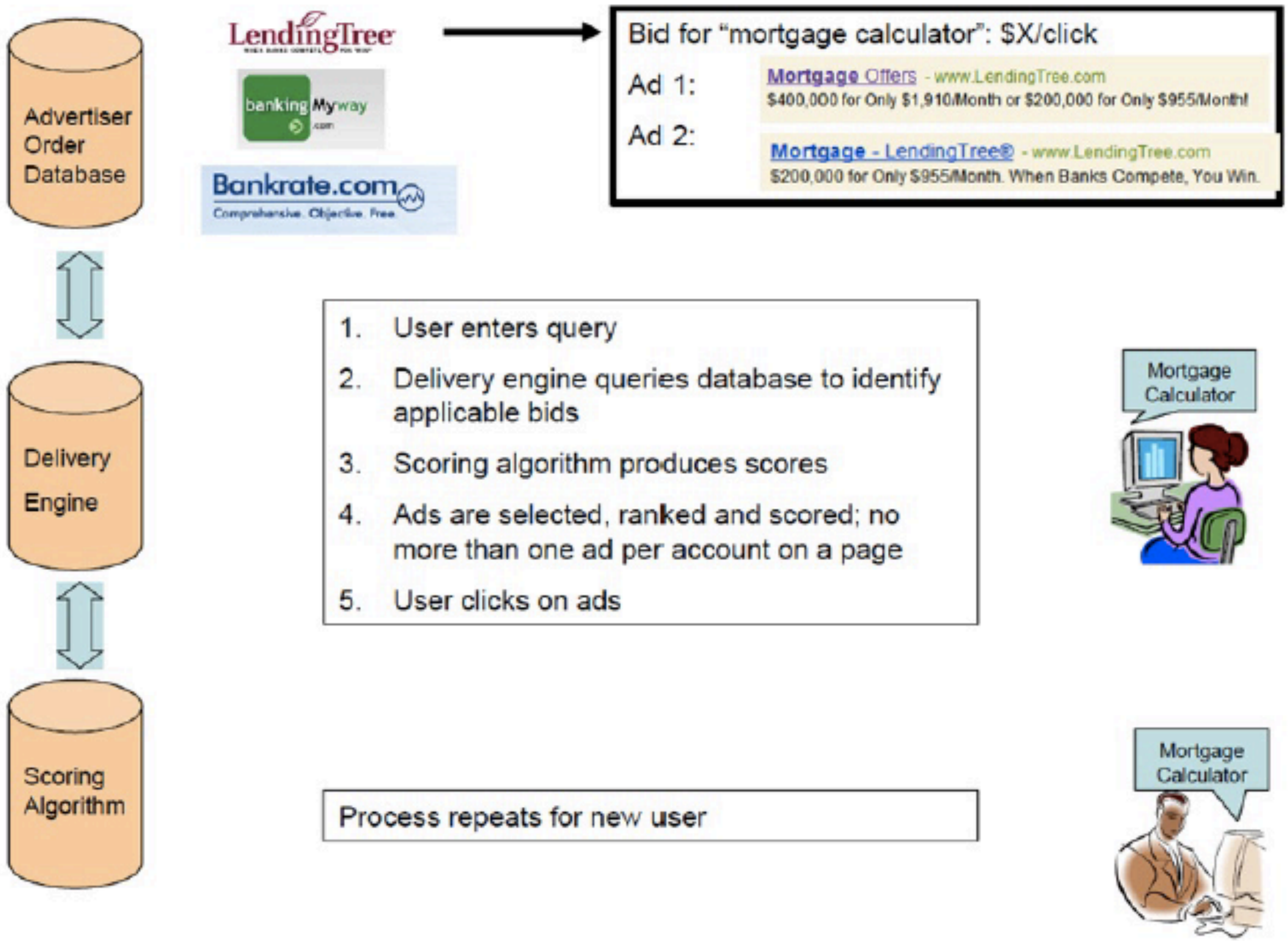
- Each bidder sets her bid to have score weighted value between marginal cost needed to decrease and increase clicks at the margin
- There are multiple Nash equilibria

Sponsored search auction

- Edelman, Ostrovsky, Schwartz (2007) show that equilibrium always exists
- There is an equilibrium where bidders pay Vickrey payoffs
- This equilibrium generates the lowest revenue to the auctioneer
- However, this auction is not truthful: bidders have incentive to shade their bids

Sponsored search auction

- In reality users arrive at high rate with little feedback to bidders



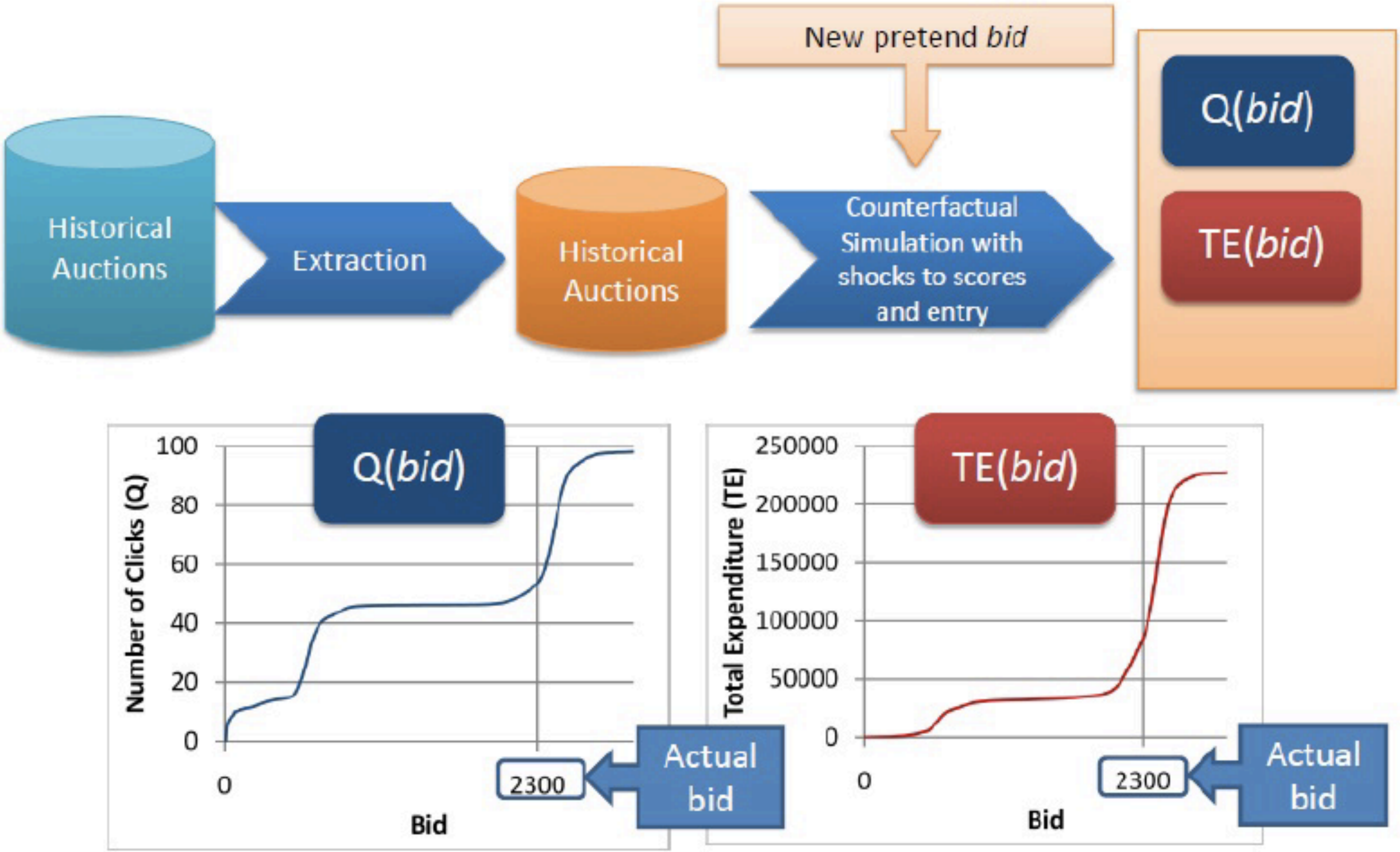
Sponsored search auction

- Bidders do not observe realization of scores
  - In fact, scores are generated by proprietary prodiction algorithm
- Can model bidders responding to expected outcome over many user queries
- Bidders characterized by values per click (VPC)
- Expected utility of bidder i is

$$\text{Utility}(\text{bid}_i;\text{VPC}_i) = \text{VPC}_i\text{Clicks}_i(\text{bid}_i) - \text{Payment}_i(\text{bid}_i)$$

  - $\text{Clicks}_i(\text{bid}_i)$ and $\text{Payment}_i(\text{bid}_i)$ are expected allocation and payment rule (with score uncertainty)

Modeling the bidders



Bid optimization

- Keep increasing the bid until marginal cost exceeds value

$\text{Utility}(\text{bid}_i;\text{VPC}_i) = \text{VPC}_i\text{Clicks}_i(\text{bid}_i) - \text{Payment}_i(\text{bid}_i)$

$$\text{VPC}_i=(\partial\text{Payment}_i(\text{bid}_i)/\partial\text{bid}_i)/(\partial\text{Click}_i(\text{bid}_i)/\partial\text{bid}_i)$$

- Note: can use similar approach if there are other objectives or there are budget constraints

Bid optimization