

The Dollar Auction game: a paradox in noncooperative behavior and escalation¹

MARTIN SHUBIK

Department of Administrative Sciences, Yale University

The Game

There is an extremely simple, highly amusing, and instructive parlor game which can be played at any party by arranging for the auction of a dollar. This game illustrates some of the difficulties with the noncooperative equilibrium concept and games in extensive form (von Neuman and Morgenstern, 1945).

The game is simplicity itself and is usually highly profitable to its promoter. The auctioneer auctions off a dollar bill to the highest bidder, with the understanding that *both* the highest bidder and the second highest bidder will pay. For example, if A has bid 10 cents and B has bid 15 cents, then the auctioneer will obtain 25 cents, pay a dollar to B, and A will be out 10 cents.

Suppose that bids must be made in multiples of 5 cents. Furthermore, suppose that the game ends if no one bids for a specific length of time. Ties are resolved in favor of the bidder closest to the auctioneer.

These rules completely specify the game except for a finite end rule; i.e., as specified, bidding could conceivably never cease. We could add an upper limit to the amount that anyone is permitted to bid. However, the analysis is confined to the (possibly infinite) game without a specific termination point, as no

particularly interesting general phenomena appear if an upper bound is introduced.

In playing this game, a large crowd is desirable. Furthermore, experience has indicated that the best time is during a party when spirits are high and the propensity to calculate does not settle in until at least two bids have been made. For the purposes of the discussion and analysis, we limit ourselves to an auctioneer and two bidders, as the basic difficulties with this game can be illustrated at this level.

Let us assume that the auction has started, A has bid 5 cents and B has raised to 10 cents. By raising to 15 cents, A stands to gain 85 cents; by standing pat, he will certainly lose 5 cents. This argument holds (with modifications on gains and losses) at any stage. In particular, a turning point in the game occurs when the bidding stands with, say, A having a bid of 50 cents and B with a bid of 45 cents. At that point, it may appear to B that he should bid 55 cents and take his chances, rather than take a certain loss of 45 cents. If B bids 55 cents, then a critical zone has been passed for the auctioneer. No matter what happens to bidding, he will always make money, as the sum of the two top bids is now larger than a dollar.

The next critical zone appears in its most spectacular form when one of the bids is at a dollar. Suppose that B had bid one dollar, and A had previously bid 80 cents. At this point, A may elect to bid \$1.05 rather than lose 80 cents with certainty. Beyond this point, both bidders

¹This research was supported by National Science Foundation Grant GS-2840.

will be losing, but still may escalate their bids in order to cut down on losses.

Once two bids have been obtained from the crowd, the paradox of escalation is real. Experience with the game has shown that it is possible to "sell" a dollar bill for considerably more than a dollar. A total of payments between three and five dollars is not uncommon.

Some Formal Analysis

Considering the auction with an auctioneer and two bidders; this can be viewed as a three person constant-sum game. Let the auctioneer be Player 1 and the bidders, players 2 and 3. The characteristic function (von Neuman and Morgenstern, 1945) is:

$$\begin{aligned} V(1) &= -95 \text{ cents, } V(2) = V(3) = 0; \\ V(1,2) &= 0, V(1,3) = 0, V(2,3) = 95 \text{ cents;} \\ V(1,2,3) &= 0. \end{aligned}$$

The auctioneer cannot prevent a loss of 95 cents to himself if the two bidders form a coalition with one bidding 5 cents and the other refraining from bidding. Any coalition involving the auctioneer and only some bidders can obtain nothing. For any size of game, the only coalition that has a positive value is the one of all bidders.

When the auction is viewed as a cooperative game, it is evident that the auctioneer is at a disadvantage. When we switch to a noncooperative analysis, the locus of the disadvantage changes to the bidders.

There is a trivial and quite unsatisfactory noncooperative equilibrium point where the first bidder bids \$1.00 as his opening bid and no one else bids. This yields a payoff of zero to all.

Another solution concept which points to a further difficulty with the equilibriums at the bid of \$1.00 is that of:

$$\text{Max-Min } (P_A - P_B),$$

where P_A and P_B are the payoffs to bidders A and B respectively.

The "max-min the difference" solution can be considered in terms of a damage exchange rate. The bidders are concerned with their relative gains or losses rather than their absolute gains or losses.

Suppose that A had opened with a bid of \$1.00. Then, for the cost of 5 cents, B can inflict damage of \$1.00 on A by bidding \$1.05. The damage exchange rate is 20 to 1. Unless there is an upper boundary to the bidding, there is no boundary to the escalation in the damage exchange rate.

On Threats and Communication

The key to the understanding of the processes at work in this game is in communication conditions. Generally in a crowd the individuals bid independently. They do not have lengthy discussions with each other. Furthermore, they do not sign agreements and specify strategies.

If it were possible to specify one's complete strategy, the first bidder would bid 5 cents and say, "If anyone else bids, I will immediately bid \$1.00 if he bids less; or I will bid 5 cents more than he, if he bids \$1.00 or more." If the other bidders believe him, then this strategy will block them from bidding and he will gain 95 cents.

If there is no formal mechanism for precommitment, we would need to specify the degree of belief of the other bidders in order to check upon the stability of the market.

In fact the bidders do not communicate directly more than their immediate bid, with no contingent statement whatsoever, except whatever might be signaled by facial expression, tone of voice, or other acts associated with bidding. In this sequential process a person is required to "put your money where your mouth is." The only communication is the bid, and the only signals are the history of bidding in the auction. There is no option to go back upon your word, as you do not have a word to go back upon.

*Game Theory, Social-Psychology,
Institutions and Escalation*

This simple game is a paradigm for escalation. Once the contest has been joined, the odds are that the end will be a disaster to both. When this is played as a parlor game, this usually happens.

Can we generalize from this formal structure to interorganization fights or international escalation? Only in a limited manner is the generalization useful. The international negotiation has communication conditions considerably different from the parlor game. Signals and quasi-commitment are possible and common.

The game theory analysis of the game in extensive form shows us that the game theory model alone does not appear to be adequate. A general description of a typical play of the parlor game shows this. Why should anyone bid in the first place? Usually, it is because of fun or desire to participate in a parlor game rather than because of individualistic analysis.² Bidding proceeds fairly briskly until the point when the sum of the two top bids is greater than a dollar, after which a look of realization comes onto the faces of many participants. There is a pause and hesitation in the group when the bid goes through the one dollar barrier. From then on, there is a duel with bursts of speed until tension builds, bidding then slows and finally peters out.

The game's play appears to depend upon virtually only the social-psychology of the players, or other unstated factors of the environment in which it is played. It is far simpler than a real auction where the bidders need to evaluate the worth of items to themselves and others. It is even simpler than the Prisoner's Dilemma, where at least the concept of a 2×2 payoff matrix must be taught.

²Technically, it is not difficult to modify the game in such a manner that two individuals are randomly selected as having bid 5 cents and 10 cents respectively, thus starting the process.

In bargaining between bureaucracies or nations, very often the negotiations are carried out by fiduciaries. Large time lags are present in the system. Furthermore, statements and explicit displays of intent concerning future behavior can be, and are, made. As much of the bargaining depends upon finding out one's own powers and wants as well as the powers and wants of the other side, the dynamics will be critically influenced by the perceptions and clarity of purpose of the negotiators.

There is no neat game theoretic solution to apply to the dynamics of the Dollar Auction, or to escalation between two nations *in abstracto*. The static game theory analysis is trivial, and although of some value, it is not enlightening concerning how to proceed from statics to dynamics.

The Dollar Auction is sufficiently simple that it may be a useful experimental game, as it contains an extremely simple aspect of escalation. Even were we to obtain clear results from such a study, it would be of only limited value in understanding escalation between nations. The latter requires a specific understanding of the mechanisms for the enforcement of agreement and the meaning of threat (Shubik, 1966). The game theoretic model for bargaining between nations must differ considerably from the Dollar Auction; and although a game theory analysis alone will probably never be adequate to explain such a process, it can serve to delimit the threat and enforcement possibilities.

REFERENCES

- Shubik, M. Towards a theory of threats. In A. Mensch (ed.), *Proceedings of a Conference Under the Aegis of the NATO Scientific Committee, Toulon, June-July 1964*. London: English Universities Press, 1966.
- von Neuman, J., and O. Morgenstern. *The Theory of Games and Economic Behavior*. Princeton, N.J.: Princeton University Press, 1945.