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Measuring the Efficiency of an FCC Spectrum Auction[†]

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We propose a method to structurally estimate the deterministic component of bidder valuations in FCC spectrum auctions, and apply it to the 1995–1996 C block auction. We base estimation on a pairwise stability condition: two bidders cannot exchange two licenses in a way that increases the sum of their valuations. Pairwise stability holds in some theoretical models of simultaneous ascending auctions under intimidatory collusion and demand reduction. Pairwise stability results in a matching game approach to estimation. We find that a system of four large regional licenses would raise the allocative efficiency of the C block outcome by 48 percent. (*JEL* D44, D45, H82, L82)

The US Federal Communications Commission (FCC) auctions licenses of radio spectrum for mobile phone service, employing an innovative simultaneous ascending auction. We study data from the 1995–1996 auction of licenses for the C block of the 1900 MHz PCS spectrum band. The C block divided the continental United States into 480 small, geographically distinct licenses. A mobile phone carrier that holds two geographically adjacent licenses can offer mobile phone users a greater contiguous coverage area. One intent of auctioning small licenses is to allow bidders, rather than the FCC, to decide where geographic complementarities lie. Bidders can assemble packages of licenses that maximize the benefits from geographic complementarities. The US practice of dividing the country into small geographic territories differs markedly from European practice, where nationwide licenses are often issued. These nationwide licenses ensure that the same provider will operate in all markets, so that all geographic complementarities are realized.

Economic theory suggests that the allocation of licenses in a simultaneous ascending auction need not be allocatively efficient. Brusco and Lopomo (2002)

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and Engelbrecht-Wiggans and Kahn (2005) demonstrate that bidders may implicitly collude through the threat of bidding wars. For example, a bidder might not add an additional license to a package to take advantage of complementarities because of threats of higher, retaliatory bids on the bidder's other licenses. For auctions of multiple homogeneous items, Ausubel and Cramton (2002) demonstrate that bidders may find it profitable to unilaterally reduce demand for licenses, similarly to a monopolist raising prices and profits by reducing supply. The concern about intimidatory collusion and demand reduction in FCC spectrum auctions is well founded. Cramton and Schwartz (2000, 2001) show that bidders in the AB block did not aggressively compete for licenses and in the later DEF block auction used the trailing digits of their bids to signal rivals not to bid on other licenses.

We provide the first structural estimate of a valuation function in an FCC spectrum auction, apart from Hong and Shum (2003). They model bidding for each license as a single-unit auction and therefore do not measure complementarities. Our estimator is based on the assumption that the allocation of licenses is pairwise stable in matches, that is, an exchange of two licenses by winning bidders must not raise the sum of the valuations of the two bidders. In our econometric model, bidder valuations are a parametric function of license characteristics, bidder characteristics, and bidder private values. We use the maximum score or maximum rank correlation estimator for matching games introduced in Fox (2010a), where the objective function is the number of inequalities that satisfy pairwise stability. Such estimators for single-agent choice problems were introduced in Manski (1975) and Han (1987). We estimate the influence of various bidder and license characteristics on bidder valuations. Finally, we compare the efficiency of the observed and counterfactual allocations of licenses and discuss the implications of our estimates for alternative auction designs.

Our estimator is consistent under an econometric version of pairwise stability in matches, which we call the rank order property. We first justify the non-econometric version of pairwise stability in matches only with references to the experimental and theoretical literatures on simultaneous ascending auctions. We then state the econometric version of pairwise stability in matches only, the rank order property, as an assumption, with the non-econometric version of pairwise stability as informal motivation.

There are three justifications of the non-econometric version of pairwise stability. In terms of the experimental literature, we use data from experimental simultaneous ascending auctions by Banks et al. (2003), where bidder valuations are known and show that the outcomes come close to satisfying pairwise stability in matches only.

Second, we analyze the outcomes generated by the equilibria in a simultaneous ascending auction discussed by Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005). In the cases they study, but allowing asymmetric bidders and licenses to some extent (as described in our Appendix), the equilibrium outcomes satisfy pairwise stability in matches only. In addition, a version of the demand reduction model of Ausubel and Cramton (2002) satisfies pairwise stability in matches only. The latter result requires straightforward bidding and no complementarities, as in Milgrom (2000).

Finally, there were few or no swaps of licenses between bidders immediately after the auction, even though such swaps were legally permissible and presumably had low transaction costs compared to the license values. Pairwise stability rules out bidders finding it profitable to trade licenses (perhaps with monetary transfers), and any swapping would be direct evidence against pairwise stability. We note that pairwise stability is a weaker condition than allocative efficiency: efficiency implies pairwise stability but not the reverse.

We contribute to the literature on spectrum auctions and the empirical analysis of multiple unit auctions in several ways. First, we structurally estimate bidder valuation functions in a spectrum auction. The existing empirical literature on FCC spectrum auctions is primarily descriptive. McAfee and McMillan (1996) provide an early analysis of the AB auction results. Cramton and Schwartz (2000, 2001) report evidence of attempts at coordination through bid signaling. Ausubel et al. (1997) and Moreton and Spiller (1998) present bid regressions showing evidence for complementarities. The structural approach is useful because it allows the researcher to quantitatively measure components of bidder valuations and the efficiency of the allocation of licenses, given the identifying assumptions.

Second, our estimator contributes to the literature on the structural estimation of multiple-unit auctions. Hortacsu and McAdams (2010); Février, Préget, and Visser (2004); Wolak (2007); Chapman, McAdams, and Paarsch (2007); and Kastl (2011) study divisible good auctions, like those for electricity and treasury bills. To our knowledge, Cantillon and Pesendorfer (2006), who study sealed-bid auctions for bus routes under package bidding, is the only other structural paper to study auctions of multiple heterogeneous items. In contrast to Cantillon and Pesendorfer, we study an ascending auction without package bidding, and we allow for implicit collusion.

All of the above papers specify a model of equilibrium behavior and invert a bidder's first-order condition to recover its valuation. Athey and Haile (2007) and Paarsch and Hong (2006) survey studies of single-unit auctions that use this strategy. This first-order-condition approach and other approaches using bid data (such as Haile and Tamer 2003) are not possible in our application because bids may be poor reflections of valuations under intimidatory collusion. None of the above estimators are consistent in the presence of implicit collusion.

Third, our paper contributes to the literature on structural estimation by allowing for a fixed effects model of unobserved heterogeneity in bidder valuations. In previous research, a maintained assumption is that the econometrician observes all publicly available information. We expect FCC bidders to have access to better information than we do. Our approach allows for license-specific fixed effects in valuations. When the first draft of this paper was circulated, the only paper that allowed for unobserved heterogeneity was Krasnokutskaya (2011). However, her approach and subsequent research rely on bid data, which is not our approach as we now explain.

Fourth, previous methods for structural estimation in auctions identify bidder valuations from final bids submitted in the auction. Theorists such as Crawford and Knoer (1981); Kelso and Crawford (1982); Leonard (1983); Demange, Gale, and Sotomayor (1986); Hatfield and Milgrom (2005); Day and Milgrom (2008); and Edelman, Ostrovsky, and Schwarz (2007), among others, have pointed out that a one-to-many, two-sided matching game is a generalization of an auction of multiple heterogeneous items. We are the first to use this insight in empirical work. We estimate the deterministic component of valuations as a function of recorded license and bidder characteristics, up to a normalization, based on the match between bidder characteristics and license characteristics. We do not use bid data in our preferred estimator. We demonstrate that a closely-related estimator that uses bid data does not yield reasonable estimates of bidder valuations. In part because we do not use bid data, we focus on estimating deterministic components of payoffs, namely how valuations relate to observed bidder and license characteristics, including the gains from geographic complementarities. We do not estimate the distribution of license- and bidder-specific private values.

Fifth, the effective size of the choice set for bidders in our application is very large. In our application, there are 480 licenses and, as a result, any estimator that relies on a direct comparison of the discrete choice between all potential packages will be computationally infeasible. Our estimator, based on pairwise stability, circumvents this computational difficulty.

Sixth, the true data generating process in a simultaneous ascending auction is a noncooperative, dynamic game. This game has multiple equilibria, including implicitly collusive and competitive equilibria. We base estimation on pairwise stability, a condition that holds across a set of equilibria in the situations studied in the theoretical literature on simultaneous ascending auctions. Pairwise stability may not hold across all equilibria and in other contexts, but it facilitates structural estimation for an otherwise intractable dynamic game. Estimation based on this arguably weak condition avoids solving for the equilibrium to the dynamic game. Computing equilibria would not be possible, given the indeterminacy of the equilibrium, the huge state space in a simultaneous ascending auction, and the massive choice set of bidders.

Finally, we estimate a two-sided, non-search matching game with transferable utility. Dagsvik (2000); Choo and Siow (2006); and Chiappori, Salanié, and Weiss (2010) work with logit-based specifications applied mostly to one-to-one matching, or marriage. We use the matching estimator of Fox (2010a).[1] An FCC bidder can win more than one license, and we focus on complementarities. We are the first paper to estimate a many-to-one matching game where the payoffs of bidders are not additively separable across licenses (unlike, say, Sørensen 2007).

We find mixed evidence concerning the efficiency of the observed allocation of the licenses. At least since Coase (1959), the use of spectrum auctions has been justified on efficiency grounds. We find that bidders strongly value complementarities between licenses and that bidders with larger initial eligibilities value licenses more. We also find that awarding each license to a distinct bidder would reduce allocative efficiency, justifying spectrum auctions as efficiency enhancing in comparison with the prior lotteries regime. However, we find evidence that the observed packages of licenses were too small for an efficient allocation given the complementarities between licenses. Indeed, we estimate that dividing the continental United States into four, large regional licenses, assortatively matched to the four largest winning bidders, would have raised the allocative efficiency of the C block by 48 percent,

[1] The empirical application in Fox (2010a) was added after the paper was initially circulated. Subsequent uses of the estimator in Fox also postdate early versions of our paper.

compared to the actual outcome. Our findings suggest that small license territories, together with the possibility of intimidatory collusion, can generate an inefficient allocation of licenses. We briefly discuss more specific policy implications.

I. Background for the C Block Auction

A. *FCC Spectrum Auctions for Mobile Phones*

Wireless phones transmit on the publicly-owned radio spectrum. In order to prevent interference from multiple radio transmissions on the same frequency, the FCC issues spectrum users licenses to transmit on specified frequencies.

There were three initial auctions of mobile phone spectrum between 1995 and 1997. The first auction (the AB blocks) sold 99 licenses for 30 MHz of spectrum for 51 large geographic regions and raised $7.0 billion for the US Treasury. The second auction (the C block) sold 493 30 MHz licenses in more narrowly-defined geographic regions to smaller bidders that met certain eligibility criteria. The C block auction closed with winning bids totaling $10.1 billion, although some bidders were unable to make payments, and their licenses were later re-auctioned. The third auction (the DEF blocks) sold three licenses for 10 MHz in each of the same 493 markets as the C block. The bids totaled $2.5 billion in the DEF blocks.

There are a number of reasons to prefer to use data from the C block auction instead of the AB or DEF blocks. First, the number of observations is much larger in the C block: there are 255 bidders in the C block compared to only 30 in the AB blocks and 155 in the DEF blocks. Furthermore, there were two licenses for sale for every geographic region in the AB blocks, and three licenses for every geographic region in the DEF blocks. An AB or DEF block winning bidder was thus guaranteed to be competing directly against at least one other winning carrier after the auction ended. This direct externality in the valuations of bidders complicates the analysis of bidding behavior. In the C block, each geographic region had only one license for sale.

The C block auction took 184 rounds, lasting from December 1995 to April 1996. Incumbent carriers did not participate in the C block because of discounts offered to small businesses. Figure 1 is a map of the licenses won by the top 12 winning bidders. The largest winner in the C block auction was NextWave, whose winning bids totaled $4.2 billion for 56 licenses, including close to $1 billion for the New York City license.

B. *After the Auction: Mergers*

C block bidders were given an extended payment plan of ten years. Many of the bidders planned to secure outside funding for both their license bids and other carrier startup costs after the auction. Many C block winners were unable to meet their financial obligations to the FCC. These new carriers were unable to secure enough outside funding to both operate a mobile phone company and pay back the FCC. Many C block winners returned their licenses to the FCC, where they were re-auctioned. Others companies merged with larger carriers (forming a large part

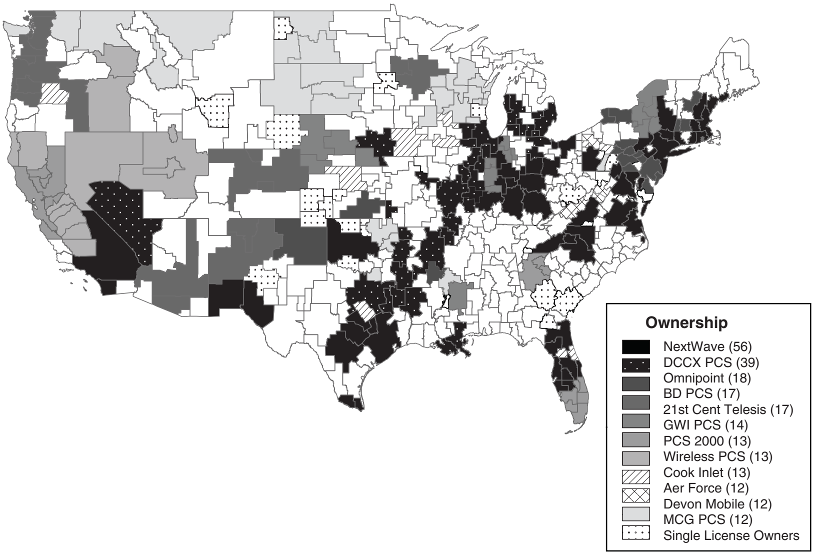

Figure 1. Map of the Licenses Won by the Top 12 Winning Bidders and Bidders Who Won Only One License

of the licenses held by T-Mobile USA, for example) or were able to protect their licenses in bankruptcy court. NextWave is the most famous case of bankruptcy protection; later it settled with the FCC and sold some of its licenses to other carriers for billions of dollars. Ex post, the C block bidders, who were accused of bidding too aggressively at the time, underpredicted the eventual market value of the licenses. However, much of this value was to larger carriers, not small-business entrants who could not secure the financing to operate as a mobile phone carrier. In 2011, only a few C block winners, such as GWI/MetroPCS, remain true independent carriers marketing service under their own brand.

The merger activity suggests that a bidder's post-auction value for winning licenses was not only a function of the package of territories it planned to serve as a mobile phone carrier. Valuations might be a function of the bidder's beliefs about the expected value from resale of its licenses, from mergers after the auction and from the risk of bankruptcy. Valuations also likely reflect the ability to serve traveling customers through roaming agreements as well as to sign up new subscribers directly. Therefore, we favor an interpretation of the estimates from our structural model that encompasses all these possibilities.

C. *Auction Rules and Bidder Characteristics*

FCC spectrum auctions are simultaneous, ascending-bid, multiple-round auctions that can take more than a hundred days to complete. A simultaneous ascending auction is a dynamic game with incomplete information. Each auction lasts multiple rounds, where in each round all licenses are available for bidding. A bidder can remain silent or enter bids to raise the standing high bids on one or several licenses. During a round, bidding on all licenses closes at the same time. The auction ends

Table 1—Characteristics of Winners and Nonwinners of Packages in the Continental United States

	Winners		Nonwinners	
Characteristic	Mean	SD	Mean	SD
Initial eligibility (millions of residents)	10.7	28.5	4.69	17.4
Revenues ($ millions)	12.8	21.7	12.4	18.8
Assets ($ millions)	39.6	67.7	40.4	72.4
Number of licenses won	5.65	7.95	0	0
Number of licenses ever bid on	40.2	73.9	13.9	41.0
Number of bidders	85		170	

when no more bids are placed on any item; bidding on all items remains possible until the end. These auction rules were designed to allow bidders to assemble packages exhibiting complementarities, while letting the bidders themselves and not the FCC determine where the true complementarities lie. If bidders have finite valuations, they will cease bidding after a finite number of rounds, although the length of the auction is not known at the start. Package bidding is not allowed; bidders place bids on each license separately.

Each bidder makes a payment before the auction begins for initial eligibility. A bidder's eligibility is expressed in units of total population. A bidder cannot bid on a package of licenses that exceeds the bidder's eligibility. For example, a bidder who pays to be eligible for 100 million people cannot bid on licenses that together contain more than 100 million residents. Eligibility cannot be increased after the auction starts. During the auction, the eligibility of bidders that do not make enough bids is reduced. By the close of the auction, many bidders are only eligible for a population equal to the population of their winning licenses.

The eligibility payments were 1.5 cents per MHz-individual in a hypothetical license for the C block. These payments are trivial compared to the closing auction prices. We use eligibility to control for a bidder's willingness to devote financial resources towards winning spectrum. This paper does not model strategic motives (such as intimidating rivals) for choosing eligibility levels. Such motives could break our assumed monotone relationship between a bidder's true valuation for licenses and its eligibility, which will make our estimates inconsistent.

Table 1 lists characteristics of the 85 winning and 170 non-winning bidders in the continental United States. The average winning bidder paid fees to be eligible to bid on licenses covering 11 million people, while the average losing bidder was eligible to bid on licenses covering only 5 million people. Bidders also had to submit financial disclosure forms (the FCC's Form 175) in order to qualify as entrepreneurs for the C block, which was limited to new entrants. Table 1 shows that the financial characteristics of winners and nonwinners were similar, which leads us to believe that these disclosure forms did not represent the true resources of bidders. Hence, in our structural estimator, we use initial eligibility as an individual bidder characteristic instead of assets or revenues.

Table 1 lists the mean number of licenses bid on and won by winners and nonwinners. The mean winning bidder won 6 licenses and entered at least one bid on

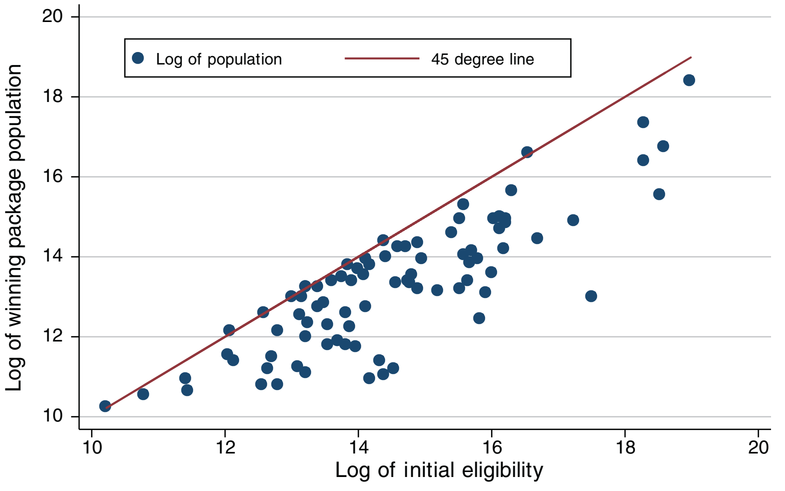

Figure 2. Log of a Winning Package's Population and the Log of the Winning Bidder's Initial Eligibility

40 licenses, compared to bidding on 14 licenses for nonwinners. Although not listed in the table, the top 15 winning bidders were active bidders on many licenses. The top 15 winners won an average of 18 licenses and bid on an average of 118 (out of 480) licenses. Most of the major winners and some of the nonwinners were investors operating on a national scale.

D. *Prices and Winning Packages*

The C block auction generated closing bids where the underlying characteristics of licenses explain much of the variation in prices across licenses. The most important characteristic of a license is the number of people living in it, who represent potential subscribers to mobile phone service. The population-weighted mean of the winning prices per resident is \$40. The second most important characteristic in determining the closing prices is population density. Spectrum capacity is more likely to be binding in more densely populated areas. A regression of a license's winning price divided by its population on its population density gives an R^2 of 0.33.[2] However, prices per resident varied widely across the AB, C, and DEF auctions. It is difficult to reconcile this across-auction variation with a view that the final bids closely reflect bidder valuations (Ausubel et al. 1997).

In the C block, the average winning bidder agreed to pay \$116 million and won a package covering 2.9 million people. The largest winner, NextWave, bid \$4.2 billion for a package covering 94 million people.

Figure 2 plots the log of a bidder's initial eligibility on the horizontal axis, and the log of the package's winning population on the vertical axis. A 45 degree line is also included; all observations lie beneath the line because a bidder cannot win

[2] Ausubel et al. (1997) use proprietary consulting data on the population density of the expected build-out areas for C block mobile phone service. They have provided us the same data, which we use here.

more than its initial eligibility. Eight winning bidders appear to be constrained; we will impose such eligibility constraints in our estimator later in the paper. The R^2 of a quadratic fit of the log of winning population to the log of initial eligibility is quite high, at 0.70. Initial eligibility is predictive of acquired spectrum.

E. *Suggestive Evidence on Complementarities*

A major justification for the simultaneous ascending auction is that it allows bidders to assemble packages of nearby licenses. Such adjacent licenses are said to exhibit complementarities or synergies. Bajari, Fox, and Ryan (2008) use data on calling plan choice to estimate that consumers do have high willingnesses to pay to avoid roaming surcharges while traveling. So there is evidence that economic primitives do justify complementarities in bidders' structural valuation functions.

One's prior might be that complementarities are not important in the spectrum auctions. The FCC chose market boundaries to be in sparsely settled areas in order to minimize complementarities across markets. Furthermore, 1900 MHz PCS wireless phone service is mainly deployed in urban areas and along major highways, so there might not even be PCS service along the boundaries of two markets. Finally, companies can coordinate with contracts (roaming agreements) if the same company does not own the adjacent licenses.

However, an initial inspection of our data is compatible with the existence of geographic complementarities. The map of the top 12 winners in Figure 1 shows several bidders win licenses in markets adjacent to each other. For example, NextWave, the largest winner, purchases clumps of adjacent licenses in different areas of the country. GWI/MetroPCS fits the cluster pattern well, winning licenses in the greater San Francisco, Atlanta, and Miami areas.

On the other hand, the majority of winning bidders win only a few licenses. Figure 1 emphasizes this by also plotting the 26 licenses in the continental United States that were the only license won by their winning bidders. Only 20 out of 85 C block winning bidders won packages of licenses where the population in adjacent licenses within the package was more than 1 million. Aer Force is the prime example of a top 12 bidder that did not seem overly concerned with complementarities. Figure 1 shows that Aer Force won 12 licenses, but that none of them are adjacent to each other. From the maps alone, it appears some winning bidders cared more about geographic complementarities than others.

Previous researchers have generally concluded that complementarities were important. Ausubel et al. (1997) and Moreton and Spiller (1998) examine whether adjacent licenses exhibited complementarities by regressing the log of winning bids on market and bidder characteristics. Ausubel et al. (1997) study the AB and C block auctions and find that the log of winning bids are positively related to whether the runner-up bidders won adjacent licenses, as one might expect in an ascending-bid auction. However, the coefficient in the C block auction is economically small, meaning that prices do not seem to strongly reflect any value of complementarities. Moreton and Spiller (1998) have better measures of incumbency and also find that winning bids are positively related to the runner-up bidder's measures of complementarities.

The previous authors also discuss scale economies, the notion that a wireless network involves fixed costs that can be spread out among more customers in a larger carrier. Scale economies can be represented by valuations convex in the total population of a package. However, because bidders with higher valuations (empirically, higher initial eligibilities) win packages with higher populations, it may be hard to empirically distinguish operating scale economies from heterogeneities in bidder valuations.

Figure 1 suggests that the clusters of nearby licenses in winning packages are possibly too small. If bidder valuations were primarily a function of complementarities, we might expect to see the entire southeast won by one bidder, for example.

The fact that many bidders win clusters of licenses suggests that complementarities matter to some degree. An alternative explanation is that a bidder has correlated license-specific values across licenses in a geographic cluster. There seems to be little scope for distinguishing between the two explanations in a spectrum auction setting. Gentzkow (2007) discusses the difficulties of distinguishing true complementarities from correlated preferences in a consumer demand setting. We assume away spatially correlated, license-specific private values and focus on complementarities because the evidence suggests that the largest winners were not local businessmen with special attachments to particular, large regions. Many of the largest winners, such as NextWave, Omnipoint, and GWI/MetroPCS, won small clusters in many regions of the country. MetroPCS has its headquarters in Dallas, but won licenses only near Atlanta, Miami, and San Francisco. DCR/Pocket won licenses stretching from Detroit to Dallas, an oddly-shaped region to be a regional specialist in. PCS2000 won mainly a cluster of licenses in the West, but its headquarters was far away in Puerto Rico. This discussion does not rule out that these bidders have spatially correlated license-specific values, but it suggests that such an explanation is not more likely than complementarities.

We do not view the price regressions of Ausubel et al. (1997) and Moreton and Spiller (1998) as a consistent estimator of bidder valuations, for at least two reasons. First, the auction induces an econometric selection problem in the final allocation of licenses to bidders. Winning packages have high payoffs for observed or unobserved reasons; otherwise they would not win. As both bidder- and license-specific valuations and complementarities across licenses contribute to total payoffs, those packages with relatively low complementarities will have relatively high bidder- and license-specific valuations. As the bidder- and license-specific valuations are typically not observed and are related to the error term in the price regression, there will be correlation between the complementary proxies and the error terms in the price regression. Linear regression will thus be inconsistent.

Even if winning packages' complementarities were somehow uncorrelated with winning packages' bidder- and license-specific valuations, the estimator would still be inconsistent. Under intimidatory collusion, as discussed in Section IF, prices will not reflect valuations and so price regressions will not identify structural parameters. In order to interpret price regressions as estimates of structural parameters, one would need to assume that the outcome to the auction is equivalent to a competitive equilibrium to the underlying economy.

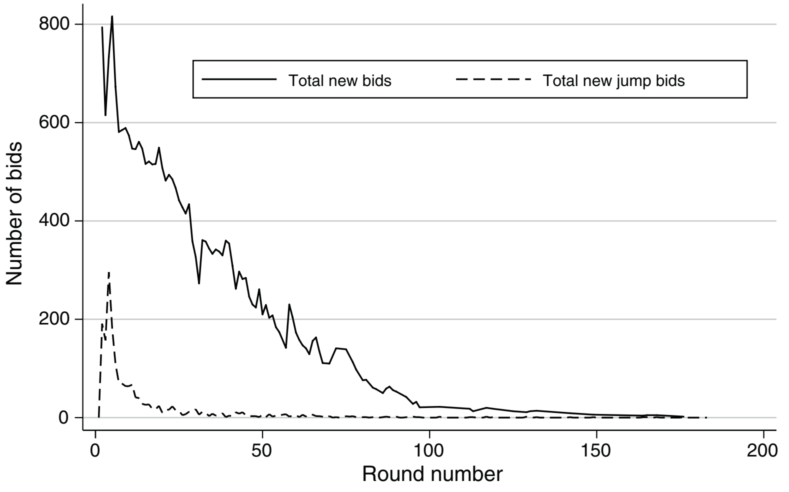

Figure 3. The Number of Jump Bids per Round

F. *Suggestive Evidence about Intimidatory Collusion*

Milgrom (2000, theorems 2,3) proves that a simultaneous ascending auction is equivalent to a tatonnement process that finds a competitive equilibrium of the economy, under two assumptions: (i) the licenses are mutual substitutes for all bidders, and (ii) all bidders bid straightforwardly. Unfortunately, neither of the assumptions needed to prove that a simultaneous ascending auction finds a competitive equilibrium appear to hold in the C block data. We have already discussed evidence that there may be complementarities in bidders' valuations.

Bidding straightforwardly means that a bidder submits new bids each period in order to maximize its structural profit function, rather than some other continuation value in a dynamic game. One violation of straightforward bidding is jump bidding. When making a jump bid, a bidder enters a bid that exceeds the FCC's minimum bid for that license and round. We define a jump bid to be any bid that is 2.5 percent greater than the minimum bid. Figure 3 shows that there was a non-trivial level of jump bidding during the C block auction.

When jump bidding, a bidder risks the chance that the jump bid will exceed the valuation of rival bidders and be the final price. A jump bidder therefore has a nonzero probability of overpaying for a license. However, there are possible strategic advantages from jump bidding. In a single unit, affiliated values model, Avery (1998) demonstrates that jump bidding may signal the jump bidder's intentions to bid aggressively throughout the auction. Because other bidders fear the winner's curse or if bidding is costly, they may stop bidding in order to avoid overpaying conditional on winning the item.

Figure 3 shows jump bidding was prevalent towards the beginning of the auction, where the risk of overpaying is much lower. Jump bids might represent signals that are attempts at intimidation, but jump bids are not evidence the signals caused other bidders to withdraw. There are anecdotes of actual retaliation. In round 3, Pocket

(DCR) placed a large jump bid of 60 percent more than the minimum for Las Vegas. In round 70, MetroPCS (GWI) outbid Pocket for Las Vegas and PCS2000 for Reno. In round 71, Pocket outbid MetroPCS on Reno and Salt Lake City, the only time Pocket bid on either of those licenses. Further, PCS2000 outbid MetroPCS on Las Vegas, the only time since round 12 that PCS2000 had bid on Las Vegas. In round 72, after seeming to retaliate against MetroPCS, Pocket enters the winning bid for Las Vegas, meaning the bid stands until the end of the auction at round 184.

There are other instances of intimidation that do not involve jump bids. Towards the end of the auction, NextWave and Aer Force were competing for Fredericksburg, VA. NextWave needed Fredericksburg to complete a regional cluster around Washington, DC. In round 162, NextWave outbid Aer Force for Fredericksburg. In round 163, Aer Force responded not only by bidding on Fredericksburg but also by bidding on Lakeland, FL. Lakeland is a small population territory that Aer Force had not bid on in a long while and that NextWave had been winning. In round 164, NextWave bid again and retook Lakeland, but never bid again on Fredericksburg. By challenging Aer Force on Fredericksburg, NextWave only succeeded in paying 10 percent (two bid increments) more to win Lakeland.

Cramton and Schwartz (2000, 2001) provide examples of signaling and implicit collusion through intimidation in the auctions for the AB and DEF blocks. We feel the evidence is strong enough that any estimation method for simultaneous ascending auction data must be based on conditions that hold in the presence of this type of implicitly collusive behavior.

II. Valuation Functions

A. *Bidders' Valuation Functions*

We now introduce the components of a bidder's profit function. There are $a = 1, \ldots, N$ bidders and $j = 1, \ldots, L$ licenses for sale. We will abuse notation and let N be the set of all bidders and L the set of all licenses. Our environment is a multiple-unit auction where bidders may win a package of licenses. We let $J \subset L$ denote such a package of licenses. In the C block, the licenses are permits to transmit mobile phone signals in specified geographic territories and there is only one license per territory. There were $N = 255$ registered bidders in the C block and 493 licenses for sale. We will limit attention to the $L = 480$ licenses for sale in the continental United States and mostly to the $H = 85$ winning bidders in the continental United States.

Bidder a maximizes its profit

$$\pi_a(J) - \sum_{j \in J} p_j$$

from winning package J at prices $(p_j)_{j \in J}$. Bidder a's profit is comprised of two parts. The term $\pi_a(J)$ is a's **valuation** for the package of licenses J and $\sum_{j \in J} p_j$ is the price that a pays for this package. In our application, we will parameterize the valuation $\pi_a(J)$ as

$$\pi_a(J) = \overline{\pi}_\beta(w_a, x_J) + \sum_{j \in J} \xi_j + \sum_{j \in J} \epsilon_{a,j}. \tag{1}$$

The function $\overline{\pi}_\beta(w_a, x_J)$ takes as arguments the characteristics w_a of bidder a and the characteristics x_J of the package of licenses J. The function $\overline{\pi}_\beta$ is parameterized by a finite vector of parameters β. Later β will be the object of estimation. The term ξ_j is a fixed effect for license j and $\epsilon_{a,j}$ is a private value specific to license j and bidder a. The fixed effect ξ_j captures the common element to the valuation of the license, such as the base contribution of population, the fact that spectrum is more scarce in more densely-populated territories and the fact that competition from incumbent carriers may be stronger in some territories than others. Let y_j be the vector of observed characteristics of license j. The characteristics x_J of a package J are formed by $x_J = \zeta(Y)$ from the $J < \infty$ license characteristics in $Y = (y_1, \ldots, y_J)$. The function ζ is known to the researcher.

In our application, we let $w_a = \text{elig}_a$ be the initial (before the auction begins) eligibility of bidder a. We treat w_a as economically exogenous, in that firms do not choose w_a for strategic reasons, such as intimidating rivals. We also assume that w_a is strictly monotone in the true preferences of bidders for spectrum; we do not allow unobserved bidder characteristics that affect the valuations of all licenses.

For license characteristics, let

$$x_J = ((\text{pop}_j)_{j=1}^{J}, \text{complem.}_J)$$

be equal to the population of all licenses in the package J, as well as a vector complem._J of proxies for the complementarities in the package. Our choice of $\overline{\pi}_\beta(w_a, x_J)$ is

$$\overline{\pi}_\beta(w_a, x_J) = \pm 1 \cdot \text{elig}_a \cdot \Big(\sum_{j \in J} \text{pop}_j\Big) + \beta' \text{complem.}_J. \tag{2}$$

The interaction $\text{elig}_a \cdot (\sum_{j \in J} \text{pop}_j)$ captures the fact, in Table 1 and Figure 2, that bidders with more initial eligibility win more licenses. We use $w_a = \text{elig}_a$ as our main measure of bidder characteristics, given that Table 1 shows financial measures are uncorrelated with winning a license. The coefficient on $\text{elig}_a(\sum_{j \in J} \text{pop}_j)$ has been normalized to ± 1, because dividing both sides of an inequality that is used in estimation by a positive constant will not change the inequality. Overall, the term $\text{elig}_a \cdot (\sum_{j \in J} \text{pop}_j)$ captures assortative matching between bidders with higher values and packages of licenses with more population.

The term $\beta' \text{complem.}_J$ provides the total contribution of the several complementarity measures in the vector complem._J. Each element of complem._J is a nonlinear construction from the characteristics of the underlying licenses in the package J. The parameters β describe the relative importance of each complementarity measure in terms of the units of $\text{elig}_a \cdot (\sum_{j \in J} \text{pop}_j)$. Overall, the term $\beta' \text{complem.}_J$ captures one-sided matching between related licenses into the same packages.

In terms of units, eligibility is the initial eligibility of a bidder. Population is just the number of residents (in the 1990 census) of the license. To aid interpretation, we divide both measures by the population of the continental United States, so that an eligibility or population of 1 corresponds to a true value of 253 million people. With this normalization, the mean population $\sum_{j \in J} \text{pop}_j$ among the 85 winning packages is 0.012 (standard deviation of 0.044), and the mean $\text{elig}_a(\sum_{j\in J} \text{pop}_j)$ is 0.0046 (standard deviation 0.030). We discuss the measures of geographic complementarities below.

We choose a simple functional form for $\overline{\pi}_\beta(w_a, x_J)$ in order to demonstrate that a parsimonious model can satisfy a high percentage of the inequalities introduced below. A more complicated functional form would have little benefit in terms of the overall fraction of inequalities that are satisfied and would obscure the interpretation of the parameters. As we do not have profit, cost, pricing, sales, and merger profits data for firms operating mobile phone carriers as a function of their coverage areas, we cannot decompose $\overline{\pi}_\beta(w_a, x_J)$ into the present discounted values of sales, marginal costs, per-period fixed costs, one-time fixed costs, and profits from merger activity.

B. *Assumptions*

We now list a series of assumptions. These assumptions are made to clarify the informational structure of the simultaneous ascending auction. They will be referenced in a series of remarks about the robustness of the theoretical results of Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005), in Appendix A. We first present assumptions about the bidder characteristics w_a.

ASSUMPTION 1: *The scalar bidder heterogeneity w_a is public information. Further, $\overline{\pi}_\beta(w_a, x_J) = h_\beta(w_a) \cdot \overline{\pi}^1_\beta(x_J) + \overline{\pi}^2_\beta(x_J)$, where $h_\beta(\cdot)$ is a monotone function, and $\overline{\pi}^1_\beta$ and $\overline{\pi}^2_\beta$ are unrestricted functions of x_J. Each w_a is in the data.*

Making w_a private information simplifies some of the analysis below, as Remark 2 in the Appendix argues. A private w_a induces ex post asymmetry in the values of licenses; a bidder with a high preference for license j_1 will often prefer license j_2 more as well. Privately observed values is the natural case to start with in developing an estimator for simultaneous ascending auctions of multiple heterogeneous items under implicit collusion. A private w_a also tracks the theoretical assumptions in the papers Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005).

The assumption of private information is inaccurate for the C block because w_a is in the data and was disclosed prior to the auction. Instead, we allow w_a to be commonly observed information. A model where w_a is public information is a model with bidders with known asymmetries. See Remarks 2, 6, and 7 in the Appendix A for discussion about the need for Assumption 1, which relates to implicit collusion in particular models. Note that regardless of whether w_a is private information or public information, w_a is a private value in the sense of Milgrom and Weber (1982), given that rival bidders would not update their own valuations if they learned the value w_a. We next turn to the ξ_j terms.

ASSUMPTION 2: *The term ξ_j is a license j fixed effect, which we assume is publicly observed by the bidders. ξ_j may be statistically dependent with y_j and hence x_J, for $j \in J$. Each ξ_j is not in the data.*

The fixed effect enters bidders' valuations additively and is meant to capture the characteristics of license j that are observed by the bidders, but not by the econometrician. For example, we lack controls for the incumbent mobile phone companies as well as the winners of the earlier AB auctions and potential merger and roaming partners. As is standard in fixed effect models, we cannot identify the effects of elements of x_J that are collinear with the fixed effects ξ_j. We can, however, identify β in $\overline{\pi}_\beta(w_a, x_J)$, which captures the interaction between the bidder and license characteristics observed by the econometrician. We will not estimate the fixed effects or their distribution, but our estimator is consistent in their presence. Our estimator will be inconsistent if the bidder heterogeneity w_a interacts with the fixed effects ξ_j.

The unobservables, $\epsilon_{a,j}$, reflect bidder a's private information about license j. We use the framework of independent private values for the $\epsilon_{a,j}$ term (any correlation occurs through w_a).

ASSUMPTION 3: *The $\epsilon_{a,j}$ are i.i.d. across bidders and licenses and are independent of all w's, x's, and ξ's. Each $\epsilon_{a,j}$ is privately observed by bidder a, but the distribution of $\epsilon_{a,j}$ is common knowledge among the bidders. Each $\epsilon_{a,j}$ is not in the data.*

These reflect bidder-specific costs and benefits from operating in a particular territory. As we discuss, our maximum rank correlation estimation approach will not allow us to identify the distribution of the $\epsilon_{a,j}$s. For the C block, the trade press and the number of licenses bid on by each bidder suggest that many winning bidders were willing to operate in any region of the country. This suggests the variance of $\epsilon_{a,j}$ is small. A small variance of $\epsilon_{a,j}$ contrasts with the AB blocks, where many bidders were incumbents trying to win territories near their existing service areas. In Section IE, we acknowledge that there is no obvious way to use data from a simultaneous ascending auction to distinguish true complementarities from spatially correlated $\epsilon_{a,j}$s. Other sources of differential values across bidders and across licenses occur through the observable ws and xs and the unobservable ξs.

The theoretical literature on the simultaneous ascending auction uses the assumption of private values. Under private values, bidders would not revise their own valuations if they were to observe the private information of rivals. For the C block, there is some evidence that some bidders stuck to their private evaluations of the value of wireless service and did not update their valuations. The bidder with the second-highest initial eligibility won no licenses because the prices exceeded that bidder's evaluation of the profit potential from wireless services.

In a common values model, ξ_j might be unobserved to the bidders as well. If a bidder was able to learn ξ_j, it would revise its valuations. Again, common values are usually not part of formal models of spectrum auctions because of technical complexity. As Hong and Shum (2003) argue empirically for the AB blocks, at the end of the auction a lot of information about ξ_j has been disclosed, possibly mitigating any winner's curse. However, this conclusion is less obvious under implicit collusion, where the link between bids and true values is imperfect.

Table 2—Winning Packages: Sample Statistics and Correlation Matrix for Geographic Complementarity Proxies

Characteristic	Mean	SD	Min	Max
Population/distance two markets in a package	0.0055	0.024	0	0.20
Trips between markets in a package in the American Travel Survey	0.0032	0.020	0	0.182
Total trips between airports in markets in a package (thousands)	0.0023	0.017	0	0.150

Correlations	Geo distribution	ATS trips
Population/distance two markets in a package	1	
Trips between markets in a package in the American Travel Survey	0.97	1
Total trips between airports in markets in a package (thousands)	0.95	0.99

Notes: The sample is the 85 winning packages in the continental United States. The formulas for these measures are equations (3) and (4).

C. *Three Proxies for Potential Complementarities*

We construct proxies for geographic economies of scope and use them as our measure of complementarities in (2). Table 2 presents descriptive statistics as well as the correlation matrix for the three measures. The measures are highly but not perfectly correlated with each other. For all geographic complementarity proxies, some fraction of the winning packages has a value of 0. For example, 26 out of the 85 winning packages contain only one license in the continental United States.

Geographic Distance.—Our first proxy for geographic scope is based on the geographic distance between pairs of licenses within a package. We measure distance between two licenses using the population-weighted centroid of each license.[3] For a package J in the set L of all licenses, potential complementarities are

$$\text{geocomplem.}_J = \sum_{i \in J} \text{pop}_i \frac{\left(\sum_{j \in J, j \neq i} \frac{\text{pop}_i \text{pop}_j}{\text{dist}_{i,j}^{\delta}}\right)}{\left(\sum_{j \in L, j \neq i} \frac{\text{pop}_i \text{pop}_j}{\text{dist}_{i,j}^{\delta}}\right)}, \tag{3}$$

where population is measured in fractions of the US total population and distance is measured in kilometers.[4] The distance, $\text{dist}_{i,j}$, between licenses i and j is, in our first

[3] The population-weighted centroid is calculated using a rasterized smoothing procedure using county-level population data from the US Census Bureau.

[4] This geographic complementarity proxy can be motivated as follows. Consider a mobile phone user in a home market i. That mobile phone user potentially wants to use his phone in all other markets. He is more likely to use his phone if there are more people to visit, so his visit rate is increasing in the population of the other license, j. The user is less likely to visit j if j is far from his home market i, so we divide by the distance between i and j. We care about all users equally, so we multiply the representative user in i's travel experience by the population of i.

set of estimates, raised to a power $\delta = 4$ to make this measure overweight nearby territories. The choice of $\delta = 4$ is arbitrary and was chosen to make the clusters of licenses seen in Figure 1 have non-trivial levels of complementarities. We also estimate the model with the choice of $\delta = 2$. The measure geocomplem.$_J$ proxies for short-distance travel and cost and marketing synergies across nearby territories. Also, the measure is similar to the well-known gravity equation in international trade. The measure also has the desirable feature that any firm's complementarities cannot decrease by adding licenses to a package.

Two Travel Measures.—Geographic measures of distance may not capture the returns to scope that concern carriers. Mobile phone customers may travel by means other than ground transportation. For example, many business users travel by air between Los Angeles and New York. In fact, the C block bidder NextWave won both the Los Angeles and New York licenses. We have two complementarity proxies based upon travel between two licenses. The first measure, from the 1995 American Travel Survey (ATS), is proportionate to the number of trips longer than 100 km between major cities. All forms of transportation are covered. The downside of this measure is that for privacy reasons the ATS does not provide enough information about rural origins and destinations to tie rural areas to particular mobile phone licenses. Our second measure, from the Airline Origin and Destination Survey for the calendar year 1994, is the projected number of passengers flying between two mobile phone license areas.[5] The drawback of the air travel measure is that it assumes all passengers stay in the mobile phone license area where their destination airport is located. We effectively code that there are zero potential complementarities between rural licenses for both travel measures. Both travel measures for a package J are population-weighted means across licenses, and take the form

$$\text{(4)} \qquad \text{travelcomplem.}_J = \sum_{i \in J} \text{pop}_i \frac{\sum_{j \in J, j \neq i} \text{trips}\,(\text{origin is } i, \text{ destination is } j)}{\sum_{j \in L, j \neq i} \text{trips}\,(\text{origin is } i, \text{ destination is } j)}.$$

Our ATS measure uses the count of raw trips in the survey, and the air travel count is inflated to approximate the total number of trips during 1994.[6] As with geographic distance, if $J = L$, travelcomplem.$_J = \sum_{i \in L} \text{pop}_i = 1$. Here again, adding a license to a package cannot take away complementarities between other licenses, so travelcomplem.$_J$ weakly increases as licenses are added to J.

[5] Intermediate stops are not counted for either dataset. For both datasets, geographic information software (GIS) was used to match origins and destinations with mobile phone licenses. For airports, the origin and destination license areas are easy to calculate. For the MSAs (Metropolitan Statistical Areas) used in the ATS, the equivalent C block license area was found using the centroid of the origin or destination MSA. The C block license boundaries for urban areas roughly follow MSAs.

[6] Our airline passenger measure does not distinguish between origins and destinations, so we simply divide the formula for the complementarity proxy by 2. If all airline trips are round trips during the same calendar year, this measure should be exactly correct.

III. Pairwise Stability

A. *Pairwise Stability and Other Properties of Auction Outcomes*

A spectrum auction is a data generating process that produces a vector of prices for each license $p^L = (p_1, \ldots, p_L)$ and an allocation of licenses $A = \{J_1, \ldots, J_N\}$. In this subsection, we define pairwise stability in matches only as well as several alternatives for comparison. This subsection does not present an equilibrium definition, instead listing properties that outcomes to auctions may or may not satisfy.

DEFINITION 1: *An allocation of bidders to licenses $A = \{J_1, \ldots, J_N\}$ satisfying $\cup_{a \in N} J_a \subseteq L$ and $J_a \cap J_b = \emptyset$ for all bidders $a \in N$ and $b \in N$ is a* pairwise stable outcome in matches only *if, for each pair of winning bidders $a \in N$ and $b \in N$, corresponding winning packages J_a and J_b, as well as licenses $i_a \in J_a$ and $i_b \in J_b$,*

$$\pi_a(J_a) + \pi_b(J_b) \geq \pi_a((J_a \backslash \{i_a\}) \cup \{i_b\}) + \pi_b((J_b \backslash \{i_b\}) \cup \{i_a\}). \tag{5}$$

Keep in mind that private values $\epsilon_{a,j}$ are included in the definition of $\pi_a(J_a)$. Pairwise stability in matches only considers swapping licenses: the total valuations of two bidders must not be increased by an exchange of one license each.[7] One way to motivate pairwise stability in matches only is to say that bidders would not want to exchange licenses along with side payments, at the end of the auction. This is a true mathematical interpretation of pairwise stability in matches only, and we will use the lack of swapping licenses after the auction to suggest that the outcome may have been pairwise stable in matches. However, when examining the output of theoretical models of simultaneous ascending auctions, we will not rely on bidders exchanging licenses with side payments. Rather, certain noncooperative equilibria to dynamic games will end up being pairwise stable in matches only.

Pairwise stability in matches only will lead to a matching approach to estimation. The results from Section 2 suggest there is important information about valuations that is contained in which bidders win which licenses. For example, the clustering of licenses in Figure 1 suggests that complementarities in licenses may be important. Table 1 and Figure 2 show that bidders with higher initial eligibilities win more licenses. This is consistent with bidders with higher eligibilities having higher valuations for licenses.

DEFINITION 2: *The outcome $(p^L, A) = (p^L, \{J_1, \ldots, J_N\})$ satisfying $\cup_{a \in N} J_a \subseteq L$ and $J_a \cap J_b = \emptyset$ for all bidders a and b is a* pairwise stable outcome in both prices

[7] One could strengthen Definition 1 to consider exchanges of bundles of two or more licenses between each of two bidders. This notion could be called "two bidders, two bundles stability." This is a stronger condition, as it implies Definition 1. The rest of the section focuses on motivating Definition 1 and not "two bidders, two bundles stability." Given the lack of motivation and the desire to use weaker rather than stronger assumptions in estimation, we focus on Definition 1. In a previous draft with a slightly different specification for the valuation function, we did estimate a model using inequalities derived from exchanges of bundles of two licenses for each bidder, and found that the point estimates were quite similar to the estimates based on Definition 1.

and matches *if, for each bidder* $a = 1, \ldots, N$, *corresponding winning package* $J_a \subset L$, *and licenses* $i \in J_a$ *and* $j \notin J_a$, $j \in L$,

$$\pi_a(J_a) - p_i \geq \pi_a((J_a \backslash \{i\}) \cup \{j\}) - p_j. \tag{6}$$

In the above definition, at the closing prices p^L, bidder a must not want to swap one of its winning licenses i for some other bidder's license j. Note that pairwise stability in both prices and matches implies pairwise stability in matches only. Adding the inequality

$$\pi_b(J_b) - p_j \geq \pi_b((J_b \backslash \{j\}) \cup \{i\}) - p_i$$

to (6) cancels the license prices and gives (5). We present estimates from estimators based on both conditions, but we focus on the weaker of the two conditions.

Because our paper seeks to structurally measure efficiency, it is important to distinguish pairwise stability in matches only from efficiency.

DEFINITION 3: *An allocation of bidders to licenses* $A = \{J_1, \ldots, J_N\}$ *is* efficient *whenever*

$$\sum_{a \in N} \pi_a(J_a) \geq \sum_{a \in N} \pi_a(J'_a)$$

for all other partitions $\{J'_1, \ldots, J'_N\}$ *of* L, *where a partition satisfies* $\cup_{a=1}^{N} J'_a \subseteq L$ *and* $J_a \cap J_b = \emptyset \ \forall a, b \in N$.

Efficiency is a stronger condition than pairwise stability in matches only. It may be efficient for one of two bidders to win all the licenses. Pairwise stability in matches only simply says an equal exchange of one license each does not raise the sum of valuations for the two bidders.

Intuitively, our estimator will measure the importance of clustering patterns and other patterns on the map in Figure 1. One insight is that this way of looking at the map can yield a consistent estimator under pairwise stability in matches only, which is weaker than efficiency. Indeed, Fox (2010b) proves that nonparametric identification of features of $\overline{\pi}(w_a, x_J)$ can occur equally as well with the conditions from pairwise stability as with the conditions from efficiency (also known as full stability). We return to nonparametric identification in Section IVC.

The definition of efficiency uses knowledge of the private values $\epsilon_{a,j}$ and, if some licenses are not allocated to bidders, fixed effects ξ_j. Our estimation strategy will not recover estimates of the distributions of these unobservables, as we have discussed. When we turn to measuring efficiency at the end of the paper, we will use the following measure of efficiency, which focuses on the contribution to valuations arising from observed license (package) and bidder characteristics.

DEFINITION 4: *An allocation of bidders to licenses* $A = \{J_1, \ldots, J_N\}$ *is* deterministically efficient *whenever*

$$\sum_{a \in N} \overline{\pi}_\beta(a, J_a) \geq \sum_{a \in N} \overline{\pi}_\beta(a, J'_a)$$

for all other partitions $\{J'_1, \ldots, J'_N\}$ *of* L, *where a partition satisfies* $\cup_{a=1}^{N} J'_a \subseteq L$ *and* $J_a \cap J_b = \emptyset \; \forall \, a, b \in N$. *Likewise,* $\sum_{a \in N} \overline{\pi}_\beta(a, J'_a)$ *for some partition* $\{J'_1, \ldots, J'_N\}$ *is a cardinal* (*non-ordinal*) measure of deterministic efficiency.

B. *Experimental Evidence on Pairwise Stability*

The rest of Section III motivates why the spectrum auction outcome satisfies pairwise stability in matches only. Banks et al. (2003) conducted experimental evaluations of the FCC simultaneous ascending auction. The authors assigned valuation functions to subject bidders and let the winning subjects keep their profits. A key advantage of experimental data is that the valuation functions of bidders are experimentally induced and hence observed in the data (Bajari and Hortaçsu 2005). We can test directly whether the auction outcome satisfied pairwise stability in matches only.

Banks et al. (2003) consider 52 auctions, each with 10 licenses for sale and between 6 and 8 bidders. In some cases, bidder valuation functions exhibited complementarities between some subset of the 10 licenses, and other times bidder valuations were additive across licenses. Within each auction, we analyzed each pair of licenses won by different bidders. We checked whether Definition 1, pairwise stability in matches only, holds for each pair of licenses. We calculate the percentage of the inequalities that are satisfied within each auction. The mean auction had 95.1 percent of its inequalities formed by the exchange of licenses between two winning bidders satisfy Definition 1. We feel that the approximation of Definition 1 to outcomes to these experimental auctions is high. Of course, the real C block auction has many more bidders and licenses than these experiments, and so the experiments cannot easily be extrapolated to the C block setting.

More ambitiously, one might be interested in the fraction of auctions where the restrictions fit the data perfectly: 100 percent of theoretically valid inequalities are satisfied. 29 out of the 52 auctions satisfy pairwise stability in matches only. In more than half of the experiments, the restrictions of pairwise stability in matches only are completely satisfied. We repeat the same exercises for pairwise stability in both matches and prices, which is Definition 2. The mean percentage of satisfied inequalities across the 52 auctions is lower than before, at 88 percent. Also, only 9.6 percent (5 out of 52) of the auctions satisfy Definition 2 perfectly: prices are such that bidders would prefer the licenses they won over alternative licenses. Thus, pairwise stability in matches only has more experimental evidence in its favor than pairwise stability in both matches and prices.

C. *Lack of Swapping Licenses after the Auction*

After the auction, reports in the trade media and government records indicate there was very little swapping of licenses. Swaps would have been legal: the FCC's unjust enrichment rules penalized transfers only to bidders that were not qualified for the C block, not swaps between C block bidders. Swapping licenses (perhaps with side payments) would have been direct evidence against the outcome being pairwise stable in matches only. While any negotiation is costly, the total bids in the C block auction were more than $10 billion, suggesting that negotiation time would be a small cost to incur in order to improve the profitability of winning bidders.

Cramton (2006) interprets the lack of immediate, post-auction resale as evidence that the C block auction's outcome is efficient, a stronger condition than pairwise stability in matches only. We think Cramton's (2006) interpretation is too strong. During the ten year period after the auction, many of the C block bidders were involved in mergers to create the large, national mobile phone carriers of today. Most of these mergers were with companies that did not directly bid in the C block auction. Fox and Perez-Saiz (2006) describe some of these mergers and show that they were primarily designed to expand the geographic coverage area of providers. The revealed preference of C block bidders to participate in mergers to increase scale is evidence that the winning packages may have been too small. Mergers are a costlier form of license adjustment than exchanges, and it is possible an outcome could be pairwise stable in matches only but inefficient, due to an inefficiently small scale for most winning bidders. Consolidation may increase valuations, but swapping licenses may not.

D. *Results of Brusco and Lopomo* (*2002*) *and Engelbrecht-Wiggans and Kahn* (*2005*)

Section IF presented suggestive evidence that bidders might have been implicitly colluding through the auction mechanism. We are not ready to conclude that there was definitely collusion. However, we believe the evidence in favor of implicit collusion is strong enough that any structural estimator for spectrum auction data should be consistent under the models of implicit collusion in simultaneous ascending auctions in the literature.

Brusco and Lopomo (2002), or BL (2002), and Engelbrecht-Wiggans and Kahn (2005), or EK (2005), present models of simultaneous ascending auctions that in many cases have equilibria where implicit collusion between bidders occurs. A common theme will be that BL's (2002) and EK's (2005) examples often satisfy pairwise stability in matches only. Note that finding symmetric, perfect Bayesian equilibrium to complex dynamic games can be challenging, as consistent sets of beliefs for all players must be found. BL (2002) and EK (2005) primarily prove theorems about what might be considered simple examples. To our knowledge, there are no general theorems about perfect Bayesian equilibria to spectrum auctions with arbitrary sets of players, licenses, and payoff structures. However, Kwasnica and Sherstyuk (2007) conduct experiments with the simultaneous ascending auction. They find that bidders' behavior shares many of the features of the BL (2002) and EK (2005) equilibria.

We consider two examples from BL (2002). There are two bidders and two licenses. Each bidder has a (privately observed) payoff π^1 for license 1, π^2 for license 2, and $\pi^{1,2} = \pi^1 + \pi^2 + k$ for licenses 1 and 2 for some $k > 0$. The vector (π^1, π^2, k) is drawn independently across the two bidders from the support $[0, 1]^2 \times [\underline{k}, \bar{k}]$.

BL (2002) first study the case of $\underline{k} = \overline{k} = 0$, or no complementarities. In BL's (2002) proposition 2, they find an equilibrium where the two bidders open with bids of 0 on the item with the higher private value. If the two bidders open with bids on different items, they split the items at a price of 0 and the auction ends. If the two bidders bid on the same item, bidding continues until the bids reach $\Delta\pi = \pi^1 - \pi^2$ for one of the two bidders. At that point, the bidder whose value of $\Delta\pi$ has been reached switches to the second item at a price of 0. The auction ends. Although not emphasized by BL (2002), the outcome of this equilibrium satisfies pairwise stability in matches only.

LEMMA 1: *In the BL* (*2002*) *equilibrium in their proposition* 2, *the outcome always satisfies pairwise stability in matches only.*

PROOF:

There are two sets of outcomes. First, the bidders a and b may split the licenses after the first round. Without loss of generality, this happens when $\pi_a^1 \geq \pi_a^2$ and $\pi_b^1 < \pi_b^2$. Addition gives

$$\pi_a^1 + \pi_b^2 > \pi_a^2 + \pi_b^1,$$

and Definition 1 is satisfied. Second, and again without loss of generality, bidder a may win license 1 after bidder b deviates to win license 2 when the price of license 1 exceeds $\Delta\pi_b = \pi_b^1 - \pi_b^2$. We thus know $\Delta\pi_b < \Delta\pi_a$. Rearranging the inequality gives

$$\pi_a^1 + \pi_b^2 > \pi_a^2 + \pi_b^1,$$

and again Definition 1 is satisfied.

It is the use of $\Delta\pi$ to decide when to switch that ensures that this implicitly collusive outcome satisfies pairwise stability in matches only. There are two reasons why this use of $\Delta\pi$ is not arbitrary. First, a bidder switching to the non-preferred license before $\Delta\pi$ would be leaving money on the table: the other bidder might drop out at $\Delta\pi - \eta$ for $\eta > 0$. The second reason is the notion of interim efficiency in Remark 5 in Appendix A. This Appendix contains a series of remarks about the robustness of the equilibrium in BL's (2002) example. Using references to explicit results in BL (2002) and EK (2005), Appendix A suggests that the existence of implicitly collusive equilibria (and to a lesser degree, outcomes that satisfy pairwise stability in matches only) may be relatively robust to the number of bidders, the number of licenses, correlation in private values for each bidder (ex post high and low types), commonly observed correlation in private values for each bidder (ex ante high and low types), ex ante asymmetries in the distribution of private values for each license, and the concern about unstudied equilibria to the model. Further, we believe pairwise stability in matches, by focusing on exchanges that keep the number of licenses won by each bidder the same, satisfies some of the spirit behind budget constraints.

In a second example, BL (2002) study the case with large complementarities, or $\underline{k} > 1$. In BL's (2002) equilibrium in their proposition 7, bidders are split into three groups. The first group has a low valuation for each of the two licenses if won separately and will never be intimidated to implicitly collude. The second group will settle for winning license 1 if the other bidder will settle for license 2, here at prices of 0. The third group will settle for license 2 if the other bidder will settle for license 1. This implicitly collusive outcome is inefficient because complementarities are large, $\underline{k} > 1$, and the outcome assigns the licenses to different bidders. BL (2002) discuss that if $\underline{k} = \bar{k}$, so that both bidders have the same value for the complementarities, then the value of the complementarities will always be competed away in competitive bidding, so that there will be no first group of bidders that refuse to implicitly collude. In our empirical specification for the valuation function (2), all complementarities will arise from the x_J term and the complementarities between different licenses for the same bidder will not be interacted with the bidder characteristic w_a.

LEMMA 2: *In the BL (2002) equilibrium in their proposition* 7, *the outcome always satisfies pairwise stability in matches only.*

PROOF:

There are two sets of outcomes. First, competitive bidding may be triggered and the bidder with the highest value, say a, for the package of both licenses will win both licenses. In this case, there are no licenses to exchange and pairwise stability in matches only has no bite. In the other outcome, bidders a and b may split the items so that, without loss of generality, a wins 1 and b wins 2. In BL's equilibrium, this happens only when $\pi_a^1 > \pi_a^2$ and $\pi_b^1 \leq \pi_b^2$. So, (5) becomes, for $i_a = 1$ and $i_b = 2$,

$$\pi_a^1 + \pi_b^2 > \pi_a^2 + \pi_b^1.$$

The equilibria BL (2002) find are natural. Having a high private-value realization for a license tells the bidder little about the valuations of its rivals. There is little to gain from bidding on a subset of licenses that are not the highest private-value realizations of the bidder. Because agents have private information, they must signal through bids to find sustainable, implicitly collusive equilibria.[8]

E. *Demand Reduction*

Demand reduction is when bidders unilaterally choose to not compete for all units they have positive valuations for. This can be profitable if they know rival bidders have decreasing returns to scale in their valuations, which can include the case of constant marginal valuations for a finite number of homogeneous items that is lower than the number of items for sale (Ausubel and Cramton 2002).

[8] The two BL (2002) examples also satisfy pairwise stability in both matches and prices. In the example in Lemma 1, if the bidders split the items at prices of 0, each bidder gets the item that gives the bidder the highest value. If instead they bid up the price on a single item, the bidder who uses $\Delta\pi$ to deviate to the second item also has a higher post-auction profit for the second item at the closing prices. Likewise, the interesting outcome in Lemma 2 has the bidders splitting the items for sale at prices of 0. Other examples may have outcomes that violate pairwise stability in matches and prices while satisfying pairwise stability in matches. In a world where licenses have asymmetric marginal distributions so that both bidders often prefer license 1 to 2, the bidders may still implicitly collude by splitting the items for sale at a price of 0 (see Remark 4 in Appendix A). This outcome is not pairwise stable in prices and matches because both bidders prefer license 1 at a price of 0. If the opportunity cost $\Delta\pi$ is used to govern which bidder coordinates on bidding on license 1 (say bid on license 1 when $\Delta\pi > c$ for some constant $c > 0$), the outcome of splitting the licenses will be pairwise stable in matches only.

Pairwise stability is an implication of the tatonnement conditions for the spectrum auction model of Milgrom (2000). In Appendix B, we use the tatonnement conditions of Milgrom (2000) to demonstrate that both Definitions 1 and 2 are satisfied in a model of demand reduction in simultaneous ascending auctions, without complementarities. The analysis of Milgrom (2000) requires straightforward bidding; strategic bidding during the auction itself is not allowed. Both BL (2002) and EK (2005) prove that competitive bidding is a Bayesian Nash equilibrium to the simultaneous ascending auction.

F. *Existence of a Pairwise Stable Allocation under Complementarities*

While the true data generating process is likely a dynamic Nash game, we rely on the conditions of pairwise stability in matches only for estimation. Milgrom (2000) and Hatfield and Milgrom (2005) give a key condition under which a competitive equilibrium, and so, a pairwise stable in matches only allocation, Definition 1, is guaranteed to exist in a many-to-one matching environment like a spectrum auction, where one bidder matches to many licenses but each license is matched to only one bidder. The key condition is that the valuation functions of bidders exhibit substitutes, not complementarities, across multiple licenses in the same package. Therefore, there is no general existence theorem for a pairwise stable allocation in a many-to-one matching environment with complementarities across multiple licenses in the same package.

Even if a model lacks a general existence theorem, it is certainly possible that the actual data are generated from a valid pairwise stable allocation. This is the maintained assumption for this paper. Not surprisingly there exists a continuum of private-value realizations where the C block satisfies pairwise stability in matches only, when $\beta = \hat{\beta}$, the estimated parameters. Looking ahead to the structural estimates, column 2 of Table 3 will indicate that 95 percent of the potential inequalities from the estimation analog of Definition 1, pairwise stability in matches only, are satisfied at the point estimates. The estimation analog does not use license-specific private values $\epsilon_{a,j}$ to fit inequalities. So the 95 percent of satisfied inequalities comes without relying on private values at all. By making private values $\epsilon_{a,j}$ for the observed matches between bidders a and licenses j high, and keeping $\epsilon_{a,j} = 0$ for matches that are not part of the final allocation, the fraction of satisfied inequalities can increase to 100 percent.

IV. The Estimator

A. *Estimator*

Fox (2010a) introduces a semiparametric maximum score or maximum rank correlation estimator for many-to-many matching games with transferable utility. Maximum score was first introduced by Manski (1975) and maximum rank correlation was introduced by Han (1987). In matching, the objective function is the same for the two estimators and the difference is whether the sample grows large in the number of markets (maximum score) or in the number of agents observed in a single market (maximum rank correlation). Our application fits into the one large market asymptotic argument. The estimator is semiparametric as no parametric distributions for the unobservables $\epsilon_{a,j}$ and the fixed effects ξ_j are imposed. The estimator is based on forming the empirical analog of the inequalities in Definition 1, which uses data on matches but not prices.

We will estimate the parameters β in (1), the valuation function. To make the econometric objective functions more readable, we will sometimes write $\overline{\pi}_\beta(a, J) \equiv \overline{\pi}_\beta(w_a, x_J)$ for bidder a and package J. Let H be the number of winning bidders. First consider a simple auction with two bidders a and b and two licenses 1 and 2. In the data, a wins 1 and b wins 2. The estimator $\hat{\beta}$ is any vector that satisfies the inequality

$$\overline{\pi}_\beta(a, \{1\}) + \overline{\pi}_\beta(b, \{2\}) \geq \overline{\pi}_\beta(a, \{2\}) + \overline{\pi}_\beta(b, \{1\}).$$

The inequality is satisfied whenever the sum of the deterministic parts of bidder valuations is not increased by an exchange of licenses. With only two bidders and two licenses, typically many such parameters β will satisfy the inequality. Any one of those parameters is a valid point estimate. Further, the confidence interval for β will be large. We need to use all of the data to produce an estimate of β with a smaller confidence interval.

For the full sample, the estimator $\hat{\beta}$ is any vector that maximizes the objective function

$$(7)\quad Q^{match}(\beta) = \frac{2}{H(H-1)} \sum_{a=1}^{H-1} \sum_{b=a+1}^{H} \sum_{i=1}^{|J_a|} \sum_{j=1}^{|J_b|} 1\big[\mathrm{pop}((J_a \backslash \{i\}) \cup \{j\}) \leq w_a, \mathrm{pop}((J_b \backslash \{j\}) \cup \{i\}) \leq w_b\big]$$
$$\cdot\, 1\big[\overline{\pi}_\beta(a, J_a) + \overline{\pi}_\beta(b, J_b) \geq \overline{\pi}_\beta(a, (J_a \backslash \{i\}) \cup \{j\}) + \overline{\pi}_\beta(b, (J_b \backslash \{j\}) \cup \{i\})\big],$$

where $\mathrm{pop}(J)$ gives the population of the package J: $\sum_{k \in J} \mathrm{pop}_k$. The objective function $Q^{match}(\beta)$ considers all combinations of two licenses won by different bidders, a and b. Only inequalities involving counterfactual packages with populations under the initial eligibility constraints for both bidders are included. If an inequality is satisfied, the count or score of correct predictions increases by 1. The estimator's inequalities include only the deterministic portion of valuations, $\overline{\pi}_\beta(w_a, x_J)$. Many inequalities will remain unsatisfied, even at the true parameter vector, because of the unobserved realizations of private values $\epsilon_{a,j}$, which also affect matches. Because not all inequalities can be satisfied, changing the score objective to squaring the deviations from deterministic pairwise stability makes the estimator inconsistent. $Q^{match}(\beta)$ is a step function and as a result, in a finite sample there can be a continuum (or multiple continua) of parameters that maximize $Q^{match}(\beta)$. Any maximizer is a consistent estimator. Reporting a 95 percent confidence region for each element of β provides a description of the estimates that encompasses the finite sample ambiguity in the point estimates.

The complete valuation function (1) includes fixed effects ξ_j. If instead we worked with $\overline{\pi}_\beta(a, J_a) + \sum_{j \in J} \xi_j$ in (7), the ξ_js would enter into both sides of (5) and difference out. Therefore, we do not need to estimate these fixed effects.

The maximum rank correlation approach only estimates the parameters β in $\overline{\pi}_\beta(w_a, x_J)$, not the distribution of any error terms. Parameters in a function of observables have always been the object of interest in maximum score (Manski 1975; Han 1987; Horowitz 1992; Matzkin 1993). We could instead write down a likelihood as the outcome to a dynamic game and attempt to estimate the distribution of unobservables. This would be difficult:

(i) There are $N \cdot L = 255 \cdot 480 = 122{,}400$ private values $\epsilon_{a,j}$ and the likelihood would be an integral over them.

(ii) The simultaneous ascending auction has multiple equilibria, including both competitive and implicitly collusive equilibria. Estimation would have to impose one equilibrium is selected.

(iii) An implicitly collusive equilibria is sustained by threats of punishment not typically seen on the path of play.

(iv) Each ξ_j would be treated as a random effect as it cannot be differenced out of the likelihood.

(v) Computing a likelihood would require evaluating all possible packages.

Several papers in the collection Cramton, Shoham, and Steinberg (2006) explore how even computing a winning bid in an alternative combinatorial auction is an active area of research in computer science. Cramton (2006) argues that a major motivation for using the simultaneous ascending auction over a package-bidding combinatorial auction is the computational challenge in evaluating all packages. Evaluating all possible packages is not a tractable estimation strategy in the C block environment.

B. *Consistency and Inference*

The asymptotics in Fox (2010a), for our application, are in the number $H \leq N$ of winning bidders observed in one large market. We assume the econometrician observes some finite number of recorded agents from an aggregately deterministic auction. Indeed, we introduce the fiction that the real-life matching market or auction with H winning bidders is a subset of some very large auction. As H gets larger in the asymptotic approximation, the researcher collects more data on a single auction.[9] The fiction is not to be taken literally; there were only 85 winning bidders in the C block.

[9] Parameter values may affect the rate at which H increases. If complementarities are large, winning packages will likely be large and H might grow slowly compared to the number of licenses. The asymptotics are in H and the speed at which H itself increases is not directly playing a role in our arguments.

We repeat the notion of pairwise stability in matches only from Fox (2010a) so readers understand the assumptions we make. Let each winning bidder have characteristics w. Likewise, each license has characteristics y. The function $\zeta(Y)$ take a set $Y = \{y_1, \ldots, y_J\}$ of $J < \infty$ license characteristics and forms a package characteristic $x = \zeta(Y)$. The function ζ is known to the researcher. The exogenous features of the matching market include $g_w(w)$, the density of bidder characteristics w, as well as $g_y(y)$, the density of license characteristics y. The terms w, x, and y can all be vectors. The equilibrium outcome in this auction includes a density $g_{x,w}^{\beta,S}(\langle w, x\rangle)$, which gives the frequency of the ordered pair $\langle w, x\rangle$, representing a winning package with characteristics x for a bidder with characteristics w. The density $g_{x,w}^{\beta,S}(\langle w, x\rangle)$ is an endogenous outcome, and so it is a function of the vector of the unknown parameters β and the unknown densities $S = (g_\epsilon(\epsilon), g_\xi(\xi \mid y))$ of both the license- and bidder-specific private values and the license fixed effects.

ASSUMPTION 4: *Let $\langle w_1, x_1\rangle$ and $\langle w_2, x_2\rangle$ be two hypothetical winning packages and let $x_1 = \zeta(Y_1)$ for $Y_1 = \{y_{1,1}, \ldots, y_{1,J_1}\}$ and $x_2 = \zeta(Y_2)$ for $Y_2 = \{y_{2,1}, \ldots, y_{2,J_2}\}$. Let $y_{1,i_1} \in Y_1$ and $y_{2,i_2} \in Y_2$. Let $x_3 = \zeta((Y_1 \backslash \{y_{1,i_1}\}) \cup \{y_{2,i_2}\})$ and $x_4 = \zeta((Y_2 \backslash \{y_{2,i_2}\}) \cup \{y_{1,i_1}\})$. Assume, for any β and S,*

$$\overline{\pi}_\beta(w_1, x_1) + \overline{\pi}_\beta(w_2, x_2) > \overline{\pi}_\beta(w_1, x_3) + \overline{\pi}_\beta(w_2, x_4)$$

if and only if

$$g_{x,w}^{\beta,S}(\langle w_1, x_1\rangle) \cdot g_{x,w}^{\beta,S}(\langle w_2, x_2\rangle) > g_{x,w}^{\beta,S}(\langle w_1, x_3\rangle) \cdot g_{x,w}^{\beta,S}(\langle w_2, x_4\rangle).$$

This assumption is also called a rank order property. The econometric version of pairwise stability in matches is a condition on the equilibrium sorting pattern. It says that if an exchange of licenses produces a lower sum of deterministic valuations, then the frequency of observing winning packages with the same characteristics as the exchange of licenses must be lower than observing winning packages with characteristics that give higher valuations. Note that the same number of licenses is won by a bidder on both sides of the inequalities in Assumption 4. We do not ask why a single bidder did not win more licenses, because bidders may split the licenses among them because of intimidatory collusion, not efficiency. Assumption 4 rules out estimation challenges involving multiple equilibria. By using data from only one auction, we condition on the equilibrium being played in that market. Also, the assumption implicitly assumes an equilibrium allocation $g_{x,w}^{\beta,S}(\langle w, x\rangle)$ exists. Fox (2010a) discusses Assumption 4 in more detail.

Given additional assumptions on the support of β, w, and x, Fox (2010a) shows that the estimator is consistent, following the original arguments of Han (1987). One part of the usual way of showing that an extremum estimator is consistent is proving that its population objective function is uniquely globally maximized at the true parameter value (Newey and McFadden 1994). For a simpler example that gives intuition for this type of estimator, consider the binary choice maximum score estimator (Manski 1975). In the binary choice model, a consumer chooses to buy a product with characteristics c whenever $c'\gamma + \mu > 0$, where c are regressors, γ are parameters to estimate, and μ is an error term. Manski gives conditions on the error term such that a rank order property holds: $c'\gamma > 0$ if and only if $\Pr(\text{buy} \mid c) > \frac{1}{2}$. The finite sample objective function with n observations on consumers indexed by i is

$$Q_n(\gamma) = \frac{1}{n} \sum_{i=1}^{n} (1[i \text{ buys in data}] \cdot 1[c'\gamma > 0] + 1[i \text{ does not buy in data}] \cdot 1[c'\gamma < 0]).$$

Using the law of iterated expectations and the law of large numbers, the population objective function is then

$$\operatorname{plim} Q_n(\gamma) = E_c(\Pr(\text{buy} \mid c) \cdot 1[c'\gamma > 0] + (1 - \Pr(\text{buy} \mid c)) \cdot 1[c'\gamma < 0]).$$

At the true value of γ, the rank order property ensures that $\Pr(\text{buy} \mid c) > \frac{1}{2}$ whenever $c'\gamma > 0$, so that the true γ globally maximizes the population objective function if some element of c has continuous support so that ties have measure zero. Continuous support also ensures that the maximum is unique; we omit the argument.

Sherman (1993) shows that the maximum rank correlation estimator is $\sqrt{H}$-consistent and asymptotically normal. The asymptotic variance matrix derived in Sherman (1993) is complex to use in that it requires additional nonparametric estimates of components that appear in the variance matrix. To avoid this complexity we use a resampling procedure known as subsampling, which is consistent under fairly weak conditions. As Politis and Romano (1994) state, essentially the only assumption needed for the validity of subsampling is that the estimator has a limiting distribution.

C. *Nonparametric Identification of Features of the Valuation Function*

Fox (2010b) proves a sequence of theorems about the nonparametric identification of features of $\overline{\pi}(w_a, x_J)$. Functional form assumptions are not required for $\overline{\pi}$; it is not known up to a finite vector of parameters β.

In this paper, we wish to identify the allocative efficiency of the observed C block allocation and counterfactual allocations, following Definition 4. Fox (2010b) does not prove that the total sum $\sum_{a \in N} \overline{\pi}(a, J_a)$ of an allocation of licenses to bidders is nonparametrically identified up to scale. Some parametric assumptions will be needed for our measurements.

The nonparametric identification theorems do provide some insights. Expression (2) says that the parametric valuation function can be decomposed into one term involving the sorting of bidders with higher valuations to packages with more population and several terms involving the geographic complementarities among a set of licenses in the same package. Theorem 5.2 in Fox (2010b) states that we can identify the sign of the first term nonparametrically, i.e., whether bidder heterogeneity is a complement to package population. We see whether assortative matching between heterogeneous bidders and package populations is more likely than anti-assortative matching in the observed allocation. Likewise, theorem 5.6 in Fox states that we can identify nonparametrically whether each proxy for complementarities really has a positive sign in valuations. More importantly, theorems 5.3 and 5.7 in Fox state that we can nonparametrically identify the relative magnitudes of each of the complementarity measures. We can identify, without any functional form assumptions, the ratio of the complementarities between bidder heterogeneity and population to the geographic complementarities between licenses. In effect, we identify the ratio of the two-sided complementarities between bidders and licenses to the one-sided complementarities between licenses in the same package.

The parametric functional form in (2) specifies the valuation function to be known up to parameters that, by the above arguments, are known to be nonparametrically identified. Thus, the extrapolation out of sample to examine the efficiency of counterfactual allocations relies on a parametric functional form whose parameters represent objects that are nonparametrically identified within the sample.

V. Main Estimates of Valuation Functions

Table 3 lists estimates of β in the valuation function, (2), from the maximum rank correlation estimator. The numbers in parentheses are 95 percent confidence intervals. Computational details are discussed in the footnote to the table. Columns 1 and 2 report baseline estimates. The number of inequalities is 13,428. As in (2), because matches are qualitative outcomes, we normalize the coefficient on $\text{elig}_a\left(\sum_{j \in J} \text{pop}_j\right)$ to be ± 1. We estimate the other parameters β separately for the $+1$ and -1 normalizations, and pick the vector with the highest number of satisfied inequalities. The results show that $+1$ is the correct point estimate. This fits the fact in Figure 2 that bidders with more initial eligibility win packages with more total population.

Column 1 includes only one proxy for geographic complementarities: geographic distance, (3). The coefficient of $\beta_{geo.} = 0.32$ means, at the furthest extrapolation, that if one bidder with the maximum eligibility of 1 were to win the entire United States (population of 1), then the also maximized complementarities (value of $1 \cdot \beta_{geo.}$) would give a total package value of $1 \cdot 1 + 0.32 \cdot 1 = 1.32$. The value from complementarities corresponds to $0.32/1.32 = 24$ percent of the total package value. Across the 85 winning packages, the standard deviation of $\text{elig}_a(\sum_{j \in J} \text{pop}_j)$ is 0.029 and the standard deviation of geocomplem._J is 0.024. Because the standard deviations are roughly the same, the coefficient estimate $\beta_{geo.} = 0.32$ implies that variation in the geographic location of licenses, geocomplem._J, is roughly $0.32/1 = 32$ percent as important in explaining the valuation of winning bidders as variation in the match between bidders with more eligibility and packages with more population, $\text{elig}_a(\sum_{j \in J} \text{pop}_j)$.[10]

Column 2 adds the two travel based complementarity measures to the specification. Now, not only do we measure the relative importance of $\text{elig}_a(\sum_{j \in J} \text{pop}_j)$ and complementarities in sorting, we see which measure of complementarities is most important. Total trips using all forms of travel has a statistically insignificant coefficient of 0.03, while the coefficient on geocomplem._J, 0.32, is similar to the

[10] The estimates ignore the fact that the bidder OmniPoint was, outside of the auction, given (by the FCC) a special pioneer license for the highly populated market of New York City. We do not include OmniPoint's license, so we do not need to make assumptions about its disposition in the counterfactual allocations in Table 5. Failing to account for OmniPoint's total package might induce small biases in the parameter estimates.

Table 3—Maximum Rank Correlation Estimates of Valuation Parameters

Column	(1)	(2)	(3)	(4)
Distance parameter δ	4		2	
Population × bidder eligibility	+1	+1	+1	+1
	Superconsistent			
Population/distance two markets in a package	0.32	0.32	1.06	0.86
	(0.31, 0.50)	(0.30, 0.47)	(0.87, 1.56)	(0.58, 1.06)
Trips between markets in a package in the American Travel Survey		0.03		−0.62
		(−0.08, 0.40)		(−0.96, −0.27)
Total trips between airports in markets in a package (thousands)		−0.16		−0.26
		(−0.37, 0.34)		(−0.51, 0.51)
Number possible inequalities	13,428			
Percent inequalities correct	0.944	0.945	0.956	0.960

Notes: The objective function was numerically maximized using differential evolution (Storn and Price 1997). More than ten runs were performed for all specifications. The reported point estimates are the best found maxima. The parentheses are 95 percent confidence intervals computed using subsampling. Subsampling uses 200 replications and 25 packages per replication (sampled without replacement). For each 25 packages, we use only the inequalities where all licenses are from the sampled packages. Subsampled confidence regions are not necessarily symmetric around the point estimate. In unreported results, we take subsets of the data by using only the inequalities corresponding to 120 out of the 480 licenses in the United States. For each license, we evaluate the valuation functions using the full winning package, whether all of the package's licenses are among the subset of 120 or not. The confidence regions from drawing licenses are similar to the regions found by drawing packages. Subsampling has not been extended to allow for spatial autocorrelation, so we do not adjust for such correlation. Parameters that can take on only a finite number of values (here ± 1) converge at an arbitrarily fast rate; they are superconsistent.

coefficient in column 1. One interpretation is that the geographic pattern of clustering reflects more than just customers wishing to make calls while traveling. Other forms of complementarities include marketing and cost-of-service synergies. The second travel measure, air travel, has a negative point estimate with a wide confidence interval that includes 0. The wide confidence regions are not surprising given the high correlation in Table 2 between the two travel measures among winning packages.[11] The upper bound of its 95 percent confidence interval of 0.34 does allow for important role for air travel synergies.[12] The standard deviation of air travel complementarities is 0.017, which is only a little smaller than, say, the standard deviation of geographic distance complementarities of 0.024. Given the similar standard deviations, the point estimates show air travel substantially reduces valuations compared to geographic distance or the composite measure of travel.

Table 3 also lists the percentage of satisfied inequalities at the point estimates, which is a measure of statistical fit. Nintey-five percent of the inequalities are satisfied. Vertical differences in bidder valuations for licenses and complementarities across licenses in the same package can explain most of the sorting patterns at the pair of licenses level.

[11] In a previous draft, we reported results using inequalities whether or not they violated the initial eligibility constraints. Using more inequalities substantially reduces the width of the confidence regions on some of the estimates.

[12] The point estimate on air travel is a lower bound on the complementarities from air travel, as air travel also appears in the ATS survey and is being double counted. Roughly 75 percent of trips in the ATS are by car; the fraction by air increases with distance.

Columns 3 and 4 use a different δ parameter in the geographic complementarities measure based on distance and population, (3). In column 3, the coefficient on geographic distance complementarities is 1.06, higher than the point estimate of 0.32 in column 1. This is not because of a change in units; the standard deviation of geographic distance complementarities for $\delta = 2$ is similar to the standard deviation for $\delta = 4$. A similar increase of the point estimate happens when comparing the point estimate on geographic complementarities in column 4 to the point estimate in column 2. The coefficients on the two travel complementarities are negative, with wide confidence intervals on air travel.[13]

VI. Estimators Using Other Inequalities

This section explores two alternative estimators that use inequalities based on different theoretical conditions for estimation. One estimator is explicitly incompatible with intimidatory equilibria where agents split the items for sale, and the other estimator uses closing prices data to explain why one license is preferred to another license by a bidder. We show that the alternative estimators generate bizarre estimates of bidder valuations.

A. *Estimates with Forced Transfers of Licenses*

Columns 1 and 2 of Table 4 consider a variant of the estimator where bidder a adds a license j to its package J without swapping the license for another. Let $\eta(j)$ be the bidder who wins license j. A corresponding inequality for a's decision not to win j involves an increase in the number of a's licenses by 1 and a decrease in the number of $\eta(j)$'s licenses by 1. Let H be the set of 85 winning bidders. The estimator is any parameter value that maximizes

$$Q^{addmatch}(\beta) = \sum_{a=1}^{H} \sum_{j=1}^{L} 1[a \neq \eta(j)] \cdot 1[\text{pop}(J_a \cup \{j\}) \leq w_a] \\ \cdot 1\left[\bar{\pi}_\beta(a, J_a) + \bar{\pi}_\beta(\eta(j), J_{\eta(j)}) \geq \bar{\pi}_\beta(a, J_a \cup \{j\}) + \bar{\pi}_\beta(\eta(j), J_{\eta(j)} \backslash \{j\})\right],$$

where $J_{\eta(j)}$ is the complete package won by the bidder that won license j. The estimator imposes the condition that a did not increase its package by one license because the sum of valuations of a and $\eta(j)$ would go down from doing so: it would be less efficient. This condition may be untenable because a may instead have not added the license j to a's package because of a fear of suffering retaliation from bidder $\eta(j)$. Therefore, maximizing $Q^{addmatch}(\beta)$ produces an inconsistent estimator under the intimidatory equilibria in Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005).

Columns 1 and 2 report a priori unreasonable estimates. In column 1, the coefficient on complementarities is implausibly large (although with a wide confidence interval). The point estimate of 6.7 shows the contribution to valuations from

[13] If estimation does not drop inequalities where counterfactual package populations violate initial eligibility constraints, the point estimates on the complementarity measures are positive with smaller confidence intervals.

Table 4—Estimators Using Other Inequalities

Type of inequalities	Transfer of 1 license		Swaps of 1 license w/prices	
	(1)	(2)	(3)	(4)
Population × bidder eligibility	+1	+1	0.36	0.36
	Superconsistent		(−0.13, 0.41)	(−0.15, 0.42)
Population/distance two markets in a package	6.7	9.8	0.12	0.12
	(−3.0, 9.2)	(−12, 14)	(−0.23, 0.15)	(−4.82, 0.15)
Trips between markets in a package in the American Travel Survey		−0.37		0.03
		(−0.49, 1.2)		(−0.81, 0.19)
Total trips between airports in markets in a package (thousands)		−0.1		−0.09
		(−0.39, 0.06)		(-0.22, 0.04)
Price (in trillions)			−1	−1
			Superconsistent	
Number possible inequalities	16,084		73,409	
Percent inequalities correct	0.950	0.953	0.913	0.914

Notes: All estimates use $\delta = 4$. See Table 3 for computational details.

complementarities is roughly 7 times the valuation from winning an equivalent amount of population (times eligibility). The coefficient in column 2 is an even larger 9.8.[14]

B. *Estimates with Prices*

Columns 3 and 4 of Table 4 report estimates using both matches and prices data. The maximum score objective function is based on Definition 2, pairwise stability in both matches and prices. When using price in addition to matches data, the estimator $\hat{\beta}$ is any vector that maximizes the objective function

$$(8) \quad Q^{price}(\beta) = \frac{1}{L^2} \sum_{i=1}^{L} \sum_{j=1}^{L} 1[\eta(i) \neq \eta(j)] \cdot 1[\mathrm{pop}((J_{\eta(i)} \backslash \{i\}) \cup \{j\}) \leq w_{\eta(i)}]$$
$$1\big[\overline{\pi}_\beta(\eta(i), J_{\eta(i)}) - \overline{\pi}_\beta(\eta(i), (J_{\eta(i)} \backslash \{i\}) \cup \{j\}) \geq p_i - p_j\big],$$

where p_i is the final, closing price of license i and $\eta(i)$ is defined above. Here, we impose the restriction that bidder $\eta(i)$ prefers to win its package $J_{\eta(i)}$ instead of winning $(J_{\eta(i)} \backslash \{i\}) \cup \{j\}$, or license j instead of i, at the closing prices to the auction. In other words, we impose the condition that the closing prices explain why bidder $\eta(i)$ won license i instead of j. Rearranging the inequality gives the inequality in Definition 2, except that like the other estimators, the private-value terms $\epsilon_{a,j}$ are not included, as is standard for maximum rank correlation estimators. Fixed effects ξ_j

[14] A previous draft included point estimates from inequalities that violate the initial eligibility constraint. The confidence intervals are smaller, typically exclude zero, and the point estimates are even larger in magnitude.

cannot be allowed in this type of estimator; for consistency using prices, we must assume the fixed effects are always zero.[15]

Akkus and Hortaçsu (2007) were the first to use the estimator with prices and perform a Monte Carlo study. In all of our Monte Carlo experiments (see Appendix C for some examples) with i.i.d. private-value terms $\epsilon_{a,j}$, the estimator performs extremely well.

In columns 3–4 of Table 4, we have included price, measured in *trillions* of dollars. The coefficient on price is normalized to be ± 1 and estimated to be -1. Taken literally, the coefficient on 0.36 on $\text{elig}_a \left(\sum_{j \in J} \text{pop}_j\right)$ in column 3 says that the value of a bidder with eligibility equal to the entire US's population winning the entire US is \$360 billion (although it is not statistically distinct from zero). Likewise, the value of complementarities from a nationwide license is \$120 billion. These estimates are absurdly high, given that the bids for the C block totaled \$10.1 billion. Indeed, the annual revenue for the wireless phone industry in 2006, with nine or more active licenses per territory (not just the C block), was \$113 billion. It is unlikely that bidders in 1996 felt the C block had 7–8 times the stock of *profit* potential as the yearly flow of *revenue* from all blocks combined 10 years later.

How is the model fitting the outcome data? Only the ratio of two parameters that enter structural payoffs linearly, say $\beta_{geo}/\beta_{price}$, is identified from an inequality. A high dollar value for non-price package and bidder characteristics is equivalent to saying the estimated coefficient on license price β_{price} would be economically small in magnitude if some other characteristic's coefficient was normalized to ± 1. A small coefficient on price is consistent with the finding in Section ID that population and population density, characteristics mostly subsumed into ξ_j, explain most price variation.

As we discussed in Section IE, Ausubel et al. (1997) included measures of the runner-up bidder's potential complementarities in a license-level price regression, and found a nonzero but economically small coefficient. Together, the estimates from (8) and the price regressions suggest that prices may not clear the market in the sense of sorting price taking bidders to different packages in a competitive market. Pairwise stability in prices and matches, Definition 2, may not be satisfied.

VII. Counterfactual Efficiencies and Policy Implications

A. *Actual and Counterfactual Deterministic Efficiencies*

We compare the efficiency of the observed allocation of licenses to that of several counterfactual license allocations. The parameter estimates from the previous section suggest that some of the various measures of complementarities are important determinants of bidder valuations. However, the auction allocated licenses to

[15] In Definition 2 and the objective function (8), ξ_j does not difference out of the inequality, like it does in Definition 1. Therefore, we have also estimated specifications including population and population density (we used all inequalities, not just those under initial eligibility). The point estimates on the covariates that affect the efficiency of alternative allocations of licenses to bidders are then \$886 billion for winning the entire US's population for eli $\text{g}_a\left(\sum_{j \in J} \text{pop}_j\right)$, and \$743 billion for the geographic complementarities geocomplem._J for winning the entire United States. These estimates dramatically reinforce the finding that, under an alternative scale normalization, the coefficient β_{price} is estimated to be economically small.

Table 5—Counterfactual Deterministic Efficiency from Five Allocations: Point Estimates Imposing Eligibility Constraints

Allocation	$\text{elig}_a(\sum_{j \in J} \text{pop}_j)$	Geographic distance	Air travel	ATS trips	Total
C block: 85 winning packages	$1 \cdot 0.39 =$ 0.39	$0.32 \cdot 0.47 =$ 0.15	$-0.16 \cdot 0.20 =$ -0.03	$0.03 \cdot 0.27 =$ 0.01	0.52
All 480 licenses won by different bidders	$1 \cdot 0.17 =$ 0.17	$0.32 \cdot 0 =$ 0	$-0.16 \cdot 0 =$ 0	$0.03 \cdot 0 =$ 0	0.17
Each 47 MTAs separate package	$1 \cdot 0.20 =$ 0.20	$0.32 \cdot 0.72 =$ 0.23	$-0.16 \cdot 0.04 =$ -0.01	$0.03 \cdot 0.17 =$ 0	0.43
Four large, regional licenses (top four of the 85 actual winners win)	$1 \cdot 0.50 =$ 0.50	$0.32 \cdot 0.96 =$ 0.31	$-0.16 \cdot 0.37 =$ -0.06	$0.03 \cdot 0.58 =$ 0.02	0.77
Nationwide license for entire United States (NextWave wins)	$1 \cdot 0.71 =$ 0.71	$0.32 \cdot 1 =$ 0.32	$-0.16 \cdot 1 =$ -0.16	$0.03 \cdot 1 =$ 0.03	0.90

Notes: Eligibility, population, and all three complementarity proxies range from 0 to 1. These counterfactuals use the point estimates from column 2 of Table 3. Only licenses in the continental United States are considered. For the 47 MTAs in the continental United States, as well as the four large regions, the top winners in the actual auction are assortatively matched to the counterfactual packages in order of population. For example, NextWave always wins the package with the highest population.

85 different bidders, which suggests that an improvement in deterministic efficiency is possible by grouping licenses into larger winning packages. Furthermore, our earlier results suggest that demand reduction and intimidatory collusion may be present in the auction, which causes or exacerbates this inefficiency.

We did not include any bidder or license characteristics in the deterministic valuation function (2) that (ex ante) would seemingly make smaller licenses optimal. Nor can we think of obvious measures in the US mobile phone industry, as this industry does seem to benefit from geographic scope, if only because of the demand side preference for larger calling areas that we estimate in Bajari, Fox, and Ryan (2008). It is more or less given that geographically larger licenses will improve deterministic allocative efficiency. The purpose of this section is to quantitatively measure exactly how much more geographically larger licenses will improve allocative efficiency. If the efficiency gain is small, there might not be much scope for policy in improving the allocation. We use the point estimates from column 2 of Table 3 and the definition of deterministic efficiency in Definition 4. The results are in Table 5.

For a given allocation of licenses, Table 5 reports the value of $\sum_{a \in N} \overline{\pi}_{\beta}(a, J_a)$. It is easiest to look at the last row of the table first. The last row considers the largest winner (and bidder with the highest initial eligibility), NextWave, winning all 480 licenses in the continental United States. NextWave was initially eligible for 176 million people, or 71 percent of the 1990 population. Therefore, the contribution to total value from NextWave's differential use for licenses is 0.71. NextWave winning all licenses would maximize the three geographic-complementarity proxies, at values of 1 each. So the total differential value (excluding the ξ_js) of a nationwide license is $1 \cdot 0.71 + \beta_1 \cdot 1 + \beta_2 \cdot 1 + \beta_3 \cdot 1$, where the three βs are the complementarity parameters estimated in column 2 of Table 3. The total value of a nationwide license is then 0.90.

Now consider the other four efficiency evaluations. The first row considers the actual allocation of bidders to licenses in the C block auction. The total surplus generated by the C block is 0.52, less than the 0.90 from the nationwide license. The terms in three of the four columns (excepting the column using the negative point estimate) are smaller than in the bottom column, suggesting that the C block failed to maximize the potential benefits from complementarities.

The second row considers an extreme where all 480 licenses are won by separate bidders. There can be no across-license complementarities. We impose that the licenses auctioned in the C block are the lowest level of disaggregation possible. There are 255 C block bidders (losers and winners). We assortatively match bidders to licenses by initial eligibility for bidders and population for licenses, so that NextWave wins New York, for example. For the $480 - 255 = 225$ licenses with the smallest populations, we say they are won by bidders with the lowest (255th) level of initial eligibility. The results show that the contribution from the $\text{elig}_a\left(\sum_{j \in J} \text{pop}_j\right)$ term is 0.17, smaller than the actual C block allocation's value of 0.39 by about half. This reflects bidders with lower valuations winning licenses.

The third row considers grouping the 480 C block licenses into 47 packages reflecting the 47 Major Trading Areas (MTAs) in the continental United States used for the 1995 AB spectrum auction. No C block license belongs to more than one MTA. The MTAs are natural groupings centered around large metropolitan areas, but including lots of rural territory as well. Again, we assortatively match winning bidders to licenses based on initial eligibility and population, so again NextWave wins New York. However, in the C block auction NextWave won New York and a lot more, so here the contribution from assortative matching between heterogeneous bidders and package population is low, at 0.20. However, the design of the MTA boundaries ensures that most local, geographic distance complementarities are captured. The measure of geographic distance complementarities rises from 0.47 to 0.72. On the other hand, the MTAs are only local areas, and so a great deal of travel between regions occurs across MTAs. The values of the travel geographic complementarity measures are small under the MTA scenario. The total value of this allocation is 0.43, lower than the actual C block allocation.

The fourth row considers splitting the United States into four large regions: the Northeast, Midwest, South, and West. We assign each of the 47 MTAs to one of these groupings. The Midwest is roughly from Pittsburgh to Wichita, Washington, DC is in the Northeast, and Oklahoma and Texas (other than El Paso) are in the South. We take the four largest winners by initial eligibility and assortatively match them to the four regions by population. NextWave's package is the Midwest; it is still slightly smaller in population than the package NextWave won in the C block. The fourth row shows that the contribution from differential bidder valuations is now higher, the measure of geographic distance complementarities is close to 1, and the two travel measures are about twice as high as the C block values. Thus, a system of four large regions raises the value from complementarities compared to the C block and significantly raises the amount of the US population won by high-value bidders. The United States is much bigger than a typical Western European nation; auctioning four licenses is workable plan that captures a large fraction of the maximum possible deterministic efficiency, 0.77 out of 0.90. These point estimates indicate that the efficiency from four large regional licenses is 0.77, which is 48 percent higher than the efficiency of 0.52 from the C block allocation. The figure of 48 percent is a lower bound on the improvement in deterministic efficiency, because the same, high value bidder could win two or more of the large, regional licenses in an actual simultaneous ascending auction with four licenses.

B. *Policy Implications for Bidder Anonymity*

In 2006, the FCC changed policies so that the bidder identities of submitted bids are now anonymous. The intention of this rule change was to limit intimidation and signaling. A previous draft of this paper addressed one mechanism of signaling other bidders (jump bids) more explicitly. Here, our policy counterfactuals suggest that the simultaneous ascending auction produced inefficiently small winning packages. If bidder anonymity is one way of reducing the scope of intimidation, then it may make the final allocation of licenses to bidders more efficient.

C. *Bidders with Overly Optimistic Beliefs*

One limitation of our revealed preferences approach to estimate bidder valuation functions is that the winning bidders may have overstated the short-term value of the licenses. All the bidders bid less than the long-term license value (compared to the high valuations for 30 MHz of 1900 MHz spectrum in the modern mobile phone industry) but the C block winners might have been overly optimistic about the short-term prospects. In this case, overly optimistic beliefs would lead to a disjunction between the estimated valuation function consistent with bidder behavior and the function a social planner focused on a short horizon might use to evaluate the efficiency of the allocation. Optimistic beliefs may have lead the large winners to devote more money to initial eligibility than losers and small winners, thus raising the the estimated economic importance of $\mathrm{elig}_a \cdot \left(\sum_{j \in J} \mathrm{pop}_j\right)$ in the valuation function, (2). Likewise, bidders may have overstated complementarities, meaning that the parameters β in (2) are too high. As we impose that the coefficient on $\mathrm{elig}_a \cdot \left(\sum_{j \in J} \mathrm{pop}_j\right)$ is normalized to 1, these competing biases may have over or underestimated the parameters of β relative to those a social planner would prefer.

D. *Competitive Scale-Reducing Economic Forces*

Intimidation and demand reduction reduce the size of winning packages and make the resulting mobile phone industry lack true national players. At least three other economic forces that are compatible with competitive bidding work in the same direction. First, bidders may have monetary budget constraints, so that financial constraints from outside of the auction make the auction outcome inefficient. See also Remark 9 in Appendix A. Second, a bidder may run down eligibility by focusing on a smaller license, and be unable to switch to a license with a larger population once the price of the smaller license becomes too expensive. Path dependence may lock a bidder into considering only substitute licenses with relatively small populations. See also Remark 10 in Appendix A. Third, the FCC's rules prevented one bidder from winning more than 98 licenses in the C and F auctions. Only the largest winner, NextWave, was anywhere close to bumping up against this constraint.

The previous descriptive literature and our bidding anecdotes in Section IF show that bid signaling did go on during the C block auction (Cramton and Schwartz 2000). However, measuring the extent or effectiveness of signaling seems difficult when these other factors also reduce winning package sizes. We note all three of these competitive reasons for inefficiently-small winning packages are consistent with larger licenses raising efficiency.

E. *Producer versus Consumer Surplus*

By "efficiency," we mean the efficiency of the allocation of licenses to bidders, from the viewpoint of bidders and not the consumers of mobile phones. As in all papers on auctions adopting a revealed preferences approach, we cannot use the outcome of the auction to identify a social planner's welfare function separate from the valuations of the bidders. In Bajari, Fox, and Ryan (2008), we did measure the preferences of consumers; we estimated a substantial willingness to pay for larger coverage areas. Here, we consider a spectrum auction with only one license per territory and only new entrant bidders, so the auction will increase the number of competitors by one in each territory regardless of which bidder wins each territory. Therefore, an outcome of the auction that results in carriers with geographically large coverage areas will not directly allow such entrants to exercise more market power than entrants with smaller coverage areas, except through offering the higher quality product of more coverage. Still, there might be other reasons why a company with a large geographic coverage area is bad for consumers, such as the exercise of market power in vertical markets (say handset provision) that might deter technological innovation. Our approach is based on bidder revealed preference and will not detect such consumer welfare losses, although we are unaware of any empirical evidence showing negative effects of geographically large coverage areas in the mobile phone industry.

VIII. Conclusions

We measure the efficiency of the outcome of an FCC spectrum auction using a structural model of the deterministic portion of bidder valuations. A spectrum auction is a complex dynamic game, with many bidders and many items for sale. The simultaneous ascending auction is theoretically susceptible to intimidatory collusion. Intimidation may result in winning packages that are inefficiently small, as bidders split the market to coordinate on paying less to the seller.

Our approach to estimation uses an econometric version of pairwise stability in matches. Pairwise stability says the sum of valuations from two winning bidders must not be increased by swapping licenses. There are four pieces of evidence suggesting that pairwise stability is likely to hold in simultaneous ascending auctions: experimental evidence, the lack of post-auction swapping, theoretical analysis of implicit collusion, and theoretical analysis of demand reduction. Intimidatory collusion is sustained by bid signaling and threats of retaliation by reverting to straightforward bidding.

We employ a matching maximum rank correlation estimator, which maximizes the number of inequalities that satisfy pairwise stability. The estimator is computationally simple as it avoids evaluating all possible counterfactual packages. Also, the estimator controls for additive, license-specific fixed effects. We estimate valuations using data on only the matches between bidders and licenses, not data on the closing prices. Indeed, we show that two alternative estimators, including one using price data, produce bizarre estimates using the C block data.

Our estimates empirically validate the FCC's focus on complementarities when designing the mechanism for allocating radio spectrum. Also, the spectrum auction itself produces a much higher surplus than awarding licenses through the FCC's prior practices, such as lotteries. However, we find that the final allocation of licenses was allocatively inefficient before considering private values. Deterministic efficiency would increase by 48 percent by awarding four large, regional licenses to the four highest-value bidders. A nationwide license would capture even more of the total deterministic efficiency. To a rough degree, our finding that splitting the United States into four large chunks raises deterministic efficiency validates the European approach of offering nationwide licenses and hence capturing all geographic complementarities, as the largest Western European countries (France, Germany, the United Kingdom) are, in terms of population, on the scale of about a fourth of the United States.

Appendix A: Remarks about Generalizations to the Main BL (2002) Example

This Appendix discusses extensions to the main BL (2002) example, where there are two bidders, two items for sale, and no complementarities. The main BL (2002) example is proposition 2 in their paper. We follow the expositional style of the theory papers BL (2002) and EK (2005), where formal theorems are proved for simple examples and extensions are discussed less formally.

Remark 1: In the remarks that follow, often the analysis of BL (2002) and EK (2005) is worried that implicit collusion, as in the BL (2002) examples in Section IIID, is not sustainable. In these cases, bidders may find the expected value (over the private values of rivals) for competing for all the licenses to be higher than implicitly colluding. Competitive bidding does not provide a concern for the estimator. BL (2002) and EK (2005) show competitive bidding is a perfect Bayesian equilibrium that will result in an efficient outcome. Efficient outcomes are automatically pairwise stable in matches only.

Remark 2: The main BL (2002) examples requires an assumption on the marginal distribution of π^1 and (because they are identically distributed) π^2. However, little is assumed about the joint distribution. Thus, the BL (2002) examples allow a bidder's private values to be correlated across licenses. There can be ex post high-value or low-value bidders. In this case, the identities of the high-value or low-value bidders are not common knowledge. Also see theorem 4 in EK (2005), which also studies the case of joint dependence between π^1 and π^2, or ex post high and low private-value bidders. Note that if our bidder heterogeneity measure w was private information and entered π^1 and π^2, it would just induce correlation between π^1 and π^2. So a private w is nested in the above analysis. A privately observed w is convenient theoretically, but does not apply to the actual C block auction, where our measure of w was disclosed to rival bidders before the auction.

Remark 3: The main BL (2002) example studies the case of two bidders and two licenses. The discussion following theorem 4 in EK (2005) states that simple implicitly collusive equilibria are possible whenever the number of licenses exceeds the number of bidders. Further, proposition 4 in BL (2002) finds a collusive equilibrium where there are more bidders than licenses. In this equilibrium, high-value bidders raise the price to weed out weak bidders, before attempting to signal and implicitly collude. All implicitly-colluding bidders must win an item for collusion to be successful. Because of the need to weed out the weak bidders, we would not necessarily expect to see very low prices in intimidatory-collusive equilibria. Indeed, the prices in the C block were not particularly low.

Remark 4: The main BL (2002) example studies the case where each license has an identical marginal distribution. Remark 3 in BL (2002) explores the case where each of the private values for licenses 1 and 2 has a known, bidder-invariant marginal distribution and $E[\pi^1] > E[\pi^2]$: license 2 on average has a lower private value π^2. This is the case in our valuation specification: package observables (to the bidders) x_J and $\sum_{j=1}^{J} \xi_j$ shift around the mean of valuations. A formal statement of BL's (2002) proposition 2 refers to their condition A, which is a condition on the marginal distribution that ensures that even a high-value type would find it profitable to implicitly collude and win one item for a low price rather than competing and winning both items. As a high-value type a does not know the privately observed values of a rival b, this is a condition on the rival b's mean private value. Remark 3 in BL (2002) states collusion can take place under different assumptions about the private value distribution, i.e., when π^1 and π^2 have different marginal distributions and $E[\pi^1] > E[\pi^2]$. A stronger condition on the distribution of each π^j is needed because the bidder who bids on the item with a lower mean private value must be induced to stick with that item and not also compete for the other license in competitive bidding. If collusion is sustainable, it follows roughly the form in the main BL (2002) example. However, the specific equilibrium outcome described in Remark 3 of BL (2002) does not necessarily satisfy pairwise stability in matches only because bidders use a multiplicative constant (arising from the particular support conditions on the private values in the example used inremark 3 of BL 2002) to modify the value of π^1 for comparison with π^2 when deciding whether to bid on item 2 instead. If instead the bidders used the opportunity cost $\Delta\pi = \pi^1 - \pi^2$ to decide whether to open bidding on items 1 or 2, pairwise stability in matches will occur. If BL's (2002) conditions for implicit collusion fail to hold, Remark 1 states competition ensues, under which pairwise stability in matches only still holds.

Remark 5: The main BL (2002) example presents just one symmetric, perfect Bayesian equilibrium to the game in question. Straightforward bidding is always a symmetric equilibrium, as Remark 1 discusses, for example. Neither BL (2002) or EK (2005) claim to find all possible symmetric equilibria to the game. It is possible that symmetric equilibria that do not satisfy pairwise stability in matches only exist (without changing the assumptions of the main BL 2002 example), although they are currently unknown. However, BL (2002) use an equilibrium property known as *interim efficiency*. Proposition 3 in BL (2002) suggests that, under additional conditions, that the outcome in the main example maximizes a "weighted sum of all types' expected surplus," where the maximization is taken over all incentive compatible allocations such that each bidder always receives one object. Thus, proposition 3 in BL (2002) uses the property of interim efficiency to suggest that, if possible, bidders would want to coordinate on the equilibrium in the main example, rather than some arbitrary, undiscovered equilibrium. Thus, the existence of other symmetric, perfect Bayesian equilibria that have yet to be found should not dissuade us from considering the equilibrium in the main example.

Remark 6: Consider a case where there are ex ante, commonly observed high and low-type bidders. For simplicity, say the payoff to bidder a from winning license j is $\pi_a^j = w_a \cdot y_j + \epsilon_{a,j}$, where here the scalar y_j is a characteristic that raises valuations, such as the population of the license. Let the standard deviation and support of the mean-zero, i.i.d. private values $\epsilon_{a,j}$ be small and let $w_a \gg w_b$ and $y_1 \gg y_2$, so that $\pi_a^1 + \pi_b^2 > \pi_a^2 + \pi_b^1$ for all realizations of $\epsilon_{a,j}$. Implicit collusion without signaling could occur: the high-type bidder a is allocated license 1 and the low-type bidder b is allocated license 2, both at prices of 0. This equilibrium could be sustained through threats of competitive bidding if π_a^1 is always sufficiently higher than π_a^2 and the loss to a from competitive bidding, the valuations of the rival $\pi_b^1 + \pi_b^2$, is sufficiently large. This type of equilibrium does not involve signaling and is based on public information (here w_a, w_b, y_1 and y_2) rather than private information, so the equilibrium strategies are dissimilar to those in the main example, although the outcomes are quite similar. The equilibrium outcome still satisfies pairwise stability in matches only, as the bidders assortatively match to licenses: high w with high y and $\pi_a^1 + \pi_b^2 > \pi_a^2 + \pi_b^1$. Thus, ex ante, commonly observed high- and low-type bidders can be compatible with pairwise stability in matches only, even without signaling. Altogether, this discussion explains Assumption 1, which says that if valuations are monotone in the scalar w_a, w can be publicly observed.[16]

Remark 7: Another case is when there are known asymmetries in the values for bidders a and b for licenses 1 and 2. For example, bidder a may be known to on average have a high private value for license 1, while bidder b may be known to on average have a high private value for license 2. Notationally, bidder-and-license-specific private values $\pi_j^a = \epsilon_{a,j}$ have commonly observed, bidder and license specific distributions $F_{a,j}$. This general notation encompasses Remark 6 as a special case. Under bidder and license specific distributions, a lot of the information on bidder

[16] Of course, one could construct similar examples (without one of the conditions $w_a \gg w_b$ and $y_1 \gg y_2$, perhaps) where the high-type bidder is assigned the low-type item. These outcomes strike us as unnatural: we cannot imagine the pre-game coordination that would lead to a high-value bidder accepting a low-value item.

idiosyncratic valuations is public and, definitionally, no longer privately observed information. Indeed, theorem 5 in EK (2005) allows the distribution of private values to vary across bidder/item pairs. In that case, there is less need to signal using the bidding mechanism to coordinate and pairwise stability in matches only may not occur. Intuitively, this type of result just applies a folk-theorem-like result to the publicly observed part of payoffs. For an equivalent of our Lemma 1 to hold, valuations must be private or valuations must be monotone in the observed heterogeneity, as in the example in Remark 6. Many conditions under which Lemma 1 does not hold, in our experience, involve aspects of valuations that are asymmetric across licenses and are not privately observed. However, Section IE argued based on institutional details that this type of extreme asymmetry was not common in the C block auction. As we have discussed, there is little evidence that the major winning bidders were local businessmen with pre-announced, bidder- and license-specific valuations for particular licenses. Further, colluding based on ex ante known bidder-and-license-specific asymmetries would not require signaling via the bidding mechanism, as in the main BL (2002) example. Section IF and the previous, descriptive empirical literature argue that there is evidence of bidders signaling each other using the bidding mechanism, rather than relying on known asymmetries in bidder valuations.

Remark 8: Related to the previous remark is the possibility of *asymmetric*, perfect Bayesian equilibria.[17] Asymmetric equilibria are not discussed in BL (2002) and EK (2005). Consider a case where $E[\pi^j]$ is high, π^j has a small, bounded support relative to $E[\pi^j]$, and π^1 and π^2 are independent. Then the equilibrium outcome, where bidder a wins license 1 at a price of 0, and bidder b wins license 2 at a price of 0, is sustainable by the threat of resorting to competitive bidding. The cost of competition is so high that the expected value of colluding is high for both bidders, even if a has a higher private value for 2 and b has a higher private value for license 1. Here the equilibrium is asymmetric because bidder a takes an action regardless of its license values. Note that this equilibrium requires no signaling: bidders divide up the items before private values are realized. The empirical evidence in the previous literature, and in Section IF, is strongly suggestive that signaling took place.[18] Thus, relying on the equilibrium refinement of symmetry as in symmetric, perfect Bayesian equilibria, seems logical for a first estimator for simultaneous ascending auctions with complementarities given the empirical evidence. By restricting attention to symmetric equilibria, we follow the theory on the simultaneous ascending auction.

Remark 9: Bulow, Levin, and Milgrom (2009) have emphasized the role of budget constraints in a much more recent spectrum auction than the C block. Pairwise stability in matches only respects one version of a monetary budget constraint: the number of matches of each bidder is the same on the left and right sides of the

[17] Note the two uses of the word "asymmetric": asymmetric equilibria here and asymmetric bidder valuations in Remark 7.

[18] One could possibly write down an asymmetric equilibrium with signaling. In that case, the empirical evidence of signaling would not be evidence in favor of the symmetry refinements used in BL (2002) and EK (2005).

inequality. Pairwise stability in matches only does not ask why one bidder did not win more licenses at the expense of a rival, only why license j_1 was won by bidder 1 and license j_2 was won by bidder 2 and not the reverse. Thus, the inequalities in pairwise stability in matches only capture some of the spirit of budget constraints. We note that almost all other estimators for auctions of a single item do not respect budget constraints at all; bids are suggested to be informative of valuations, not budget constraints.[19]

Remark 10: FCC spectrum auctions have eligibility rules. At the end of the C block auction, all but two bidders, who were competing for a single license, had settled on their final, winning packages. Their final-round eligibilities were only slightly above the populations of their winning packages. The condition of pairwise stability in matches only is not a statement about the behavior of bidders at the auction's final round, when they had no free eligibility. Rather, pairwise stability in matches only is a condition on the entire data generating process (all the rounds of the auction) and the final allocation that results from the data-generating process. The interesting signaling behavior in the BL (2002) and EK (2005) models arise at the start of the auction, when bidders' eligibilities are above the populations of their final winning packages.

Remark 11: An additional concern in simultaneous ascending auctions is the exposure problem, where a bidder fails to secure additional licenses to complete a package, and therefore the bidder prefers to not win a license it did win at the end of the auction. Cramton (2006) argues that the price discovery advantages of and the withdrawal options in the FCC's simultaneous ascending auction design mitigate any exposure problem. Pairwise stability in matches will still hold under an exposure problem if valuations would not be increased by swapping licenses. Given the exposure problem, pairwise stability holds if the bidders are exposed on the "best of a bad menu" of licenses.

Appendix B: Demand Reduction and Pairwise Stability, without Complementarities

Demand reduction is studied by Ausubel and Cramton (2002) for the case of sealed bid auctions of multiple homogeneous items. In a simultaneous ascending auction, demand reduction is consistent with straightforward bidding by forward-looking agents. Kagel and Levin (2001) and List and Lucking-Reiley (2000) find substantial demand reduction in experiments. This section considers demand reduction, but in a market without complementarities because of a need to refer to a Milgrom (2000) theorem. Because complementarities are the focus of our empirical work, we place this material in an Appendix, although we feel the results are interesting for the estimation method. Also, Milgrom requires bidders

[19] One estimation approach would be to impose pairwise stability in matches only for exchanges of licenses with similar closing prices. Our experiments show that this reduces the empirical power (increases the width of the confidence intervals) of the estimator considerably.

Table B1—Valuations for Two-Bidder Examples of Demand Reduction

	Bidder a	Bidder b, case 1	Bidder b, case 2
License 1	$\pi_a^1 \geq \pi_a^2$	$\pi_b^1 \leq \pi_a^1$, $\pi_b^1 \leq \pi_b^2$	$\pi_b^1 \leq \pi_a^1$, $\pi_b^1 \geq \pi_b^2$
License 2	$\pi_a^2 \leq \pi_a^1$	$\pi_b^2 \leq \pi_a^2$, $\pi_b^2 \geq \pi_b^1$	$\pi_b^2 \leq \pi_a^2$, $\pi_b^2 \leq \pi_b^1$
Both 1 and 2	$\pi_a^{1,2} = \pi_a^1 + \pi_a^2$	$\pi_b^{1,2} = \max\{\pi_b^1, \pi_b^2\}$	$\pi_b^{1,2} = \max\{\pi_b^1, \pi_b^2\}$

to bid straightforwardly, so this analysis does not distinguish between publicly and privately observed information and so does not work with Bayesian Nash equilibria. On the other hand, Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005) show that competitive bidding is a Bayesian Nash equilibrium to simultaneous ascending auctions.

Consider bidders a and b competing for two licenses 1 and 2. Use the shorthand notation $\pi_a^{1,2}$ for $\pi_a(\{1, 2\})$. Let the valuations of bidders a and b for the three possible packages be as listed in Table B1, case 1. Bidder a has a higher value for all packages. Bidder b has decreasing returns to scale: there is no incremental value from winning both licenses.

If both bidders bid straightforwardly in a simultaneous ascending auction, and ignoring minimum bid increments, a will win both licenses at prices equal to b's values: $p_1 = \pi_b^1$ and $p_2 = \pi_b^2$. However, if a reduces its demand and lets b win item 2 at $p_2 = 0$, a can win item 1 at $p_1 = 0$. Bidder b accepts this because it has a demand for only one license and prefers 2 to 1. The demand reduction outcome is inefficient: valuations are maximized by having a win both items. However, when a wins 1 and b wins 2, $\pi_a^1 + \pi_b^2 > \pi_a^2 + \pi_b^1$, so that the sum of valuations cannot be increased with license swaps. Definition 1 is satisfied as the bidders disagree on the valuation ranking of the licenses. One can use the zero prices to show Definition 2 is satisfied as well.

Now we will argue that the example does not rely on bidder disagreement over the valuation ranking. Case 2 in Table B1 changes b's valuations so that a and b agree on the valuation ranking of licenses 1 and 2: $\pi_b^1 \geq \pi_b^2$. At the beginning of the auction, with $p_1 = p_2 = 0$, bidder b will bid on item 1 as b prefers 1 and has a demand for only one item. Only at a price $p_1^\star$ such that $\pi_b^1 - p_1^\star = \pi_b^2$ will b accept winning license 2 instead of 1. If $\pi_a^1 - p_1^\star > \pi_a^2$, then substituting in $p_1^\star = \pi_b^1 - \pi_b^2$ to $\pi_a^1 - p_1^\star > \pi_a^2$ again gives $\pi_a^1 + \pi_b^2 > \pi_a^2 + \pi_b^1$. Definitions 1 and 2 are satisfied.

What if in case 2, $\pi_a^1 - p_1^\star = \pi_a^1 - (\pi_b^1 - \pi_b^2) < \pi_a^2$? If a finds it profitable to reduce its demand, a will reduce its demand on license 1 and win 2, leaving $\pi_a^2 + \pi_b^1 > \pi_a^1 + \pi_b^2$. Again, $p_1^\star$ is set, by straightforward bidding, to make a and b coordinate on a pairwise stable outcome. Definitions 1 and 2 are satisfied. The points made in this example are more general.[20]

[20] The conditions for Milgrom's tatonnement process theorem rule out complementarities, in part to avoid the exposure problem. Definition 1 requires only that sum of valuations not be raised by swapping licenses. It is compatible with many forms of the exposure problem. See Remark 11 in Appendix A.

LEMMA 3: *Consider straightforward bidding in a simultaneous ascending auction with demand reduction. Under the tatonnement conditions of Milgrom* (2000), *the outcome is a pairwise stable outcome to a matching game where the maximum number, or quota, of licenses that a bidder can win is the number of licenses the bidder won in the outcome. Both Definitions* 1 *and* 2 *are satisfied.*

PROOF:

Let the allocation portion of the demand reduction outcome be A, and let bidder a's wining package be J_a. For all bidders a, redefine a's valuation for a package J to be negative infinity if J has more licenses than J_a. $\pi_a(J) = -\infty$ for $|J| > |J_a|$. Then Milgrom's (2000) tatonnement process theorems (theorems 2 and 3 in Milgrom 2000) show that the simultaneous ascending auction will find a competitive equilibrium (core outcome) of the economy with the truncated valuation functions. Pairwise stability, Definition 2, is implied by being in the core of the economy with truncated valuation functions. As the swaps considered in Definition 2 do not change the number of licenses won by any bidder, the valuations under the swaps are the same as under the pre-truncated valuation functions. So the outcome is pairwise stable under a matching game where bidders cannot add additional licenses to their package.

Under demand reduction, the outcome may not be efficient, but there is no reason to believe that there exist swaps of licenses that would raise sums of valuations. The lemma does not explain how much demand reduction will go on: the unilateral incentive to reduce demand requires knowledge that another bidder has strong decreasing returns to scale.[21]

Appendix C. Monte Carlo for Estimator with Both Matches and Price Data

Fox (2010a) presents Monte Carlo studies showing that the finite-sample performance of the maximum score estimator, using matches only, is reasonable. However, for a small number of bidders and licenses and a high variance of the error term, the estimator that uses data only on matches can have high bias and root mean squared error (RMSE) in a finite sample, as random noise from the $\epsilon_{a,j}$ terms dominates the matching, leaving little signal in the sorting pattern seen in the data. Like similar results in Akkus and Hortacsu (2007), Table C1 reports results from a Monte Carlo study from a one-to-one, two-sided matching market. Each bidder a matches to at most one license j, and the payoff of a bidder is $\overline{\pi}_\beta(a,j) + \epsilon_{a,j} = x_{1,a}x_{1,j} + \beta x_{2,a}x_{2,j}$. There are two characteristics for bidders and two for licenses, with characteristics for each side distributed as a bivariate normal with means $(10, 10)$, variances $(1, 1)$ and covariance 0.5. The errors are i.i.d. normal with standard deviations listed in the table. For each auction we draw observable characteristics and unobservable error

[21] The initial eligibilities of other bidders are known before bidding starts. Therefore, some forms of decreasing returns are public knowledge. Further, Cramton (2006) interprets the purchase of spectrum in a small, quick, post-auction sale (a bidder did not make its payments) by NextWave as evidence that NextWave was reducing its demand during the auction.

Table C1—Maximum Score Monte Carlo: Comparing Using Data on Only Matches to Data on Both Matches and Prices Under Tatonnement Assumptions with Noise-Dominating Matches, True Value is 1.5

Number of bidders	Number of licenses per auction	Number of spectrum auctions	Error SD	Matches		Matches + Prices	
				Bias	RMSE	Bias	RMSE
30	30	1	1	0.587	1.93	0.005	0.03
10	10	10	1	0.330	1.05	0.009	0.07
30	30	1	5	1.22	4.22	0.02	0.09
10	10	10	5	1.69	7.36	−0.02	0.446

terms and compute an equilibrium assignment and vector of prices using the primal and dual linear programs for two-sided matching (Koopmans and Beckmann 1957; Shapley and Shubik 1971). The true β is 1.5. The example is chosen to make using only matches look bad: there is not much signal about $\overline{\pi}_\beta$ in the sorting patterns if the realized matches are visually plotted in characteristic space, especially in the second half of the table where the standard deviation of $\epsilon_{a,j}$ is five times higher than in the upper part of the table. Note that, for the C block, the map in Figure 1 shows that there are clear sorting patterns; this Monte Carlo study makes using match data bad to show the potential advantages of using price data. The finite-sample bias and RMSE are always much lower with continuous transfer data, even though the data on matches alone are uninformative. For all four cases the absolute value of the bias is small for small samples, and for three of the four cases the RMSE is low compared to the true value of 1.5.

Table C1 shows a major advantage of using price data: the finite-sample performance is much better if prices are generated from a tatonnement process. There are several advantages to using only match data, even if the prices are generated by a tatonnement process. This first is transparency: there is only one type of dependent variable, so inferring parameters from the US map of winning bidders is straightforward. With two types of dependent variables, it is not as clear where identification arises from. The second is robustness. In this paper, we review models where prices are not generated by a tatonnement process, but the matches are still robust to pairwise swaps.

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